1. (a) True, by Theorem 4.9.
   (b) True, by Theorem 4.9.
   (c) False, e.g., $A_{TM}$ is not Turing-recognizable.
   (d) False. A TM $M$ may loop on input $w$.
   (e) True. List the strings in lexicographic order.
   (f) False. Homework 9, problem 1.
   (g) True, by Theorems 3.13 and 3.16.
   (h) False, by Theorem 5.4.
   (i) True, by Theorem 4.5.
   (j) False, by Corollary 4.23.

2. (a) Yes, because each element in $A$ maps to a different element in $B$.
   (b) No, because nothing in $A$ maps to $4 \in B$.
   (c) No, because $f$ is not onto.
   (d) A language $L_1$ that is Turing-recognizable has a Turing machine $M_1$ that may loop forever on a string $w \notin L_1$. A language $L_2$ that is Turing-decidable has a Turing machine $M_2$ that always halts.
   (e) An algorithm is a Turing machine that always halts.

3. (a) $q_10\#11 \ xq_30\#11 \ x0q_3\#11 \ x0\#q_511 \ x0q_6\#x1 \ xq_70\#x1 \ q_7x0\#x1 \ xq_10\#x1 \ xxq_2\#x1 \ xx\#q_4x1 \ xx\#xq_41 \ xx\#x1q_{reject}$
   (b) $q_10\#0 \ xq_20\#x \ x\#q_40 \ xq_6\#x \ q_7x\#x \ xq_1\#x \ x\#q_8x \ x\#xq_8 \ x\#x \ xq_{accept}$

4. This proof is given on slides 4-39 and 4-40. Write out the strings in $\Sigma^*$ in lexicographic order as $s_1, s_2, s_3, \ldots$. Let $B$ be the set of infinite binary sequences, which we know is uncountable from HW 9, problem 1. Define the mapping $\chi : L \rightarrow B$ such that for any language $A \in L$, we have $\chi(A)$ is the infinite binary sequence in which the $i$th bit is 1 if and only if $s_i \in A$. The mapping $\chi$ is one-to-one since if $A_1$ and $A_2$ are two different languages, then they differ by at least one string $s_j$, so then $\chi(A_1)$ and $\chi(A_2)$ differ in the $j$th bit. The mapping $\chi$ is onto since if $b = (b_1, b_2, b_3, \ldots) \in B$ is an infinite binary sequence, then if we define the language $A_b$ so that the string $s_i \in A_b$ if and only if $b_i = 1$, then $\chi(A_b) = b$. Since $\chi$ is both one-to-one and onto, $\chi$ is a correspondence. Thus, $L$ is the same size as $B$, so $L$ is uncountable since $B$ is uncountable.
5. Define the language as

\[ EQ_{\text{REX}} = \{ \langle R, S \rangle \mid R \text{ and } S \text{ are regular expressions with } L(R) = L(S) \}. \]

Recall that the proof of Theorem 4.5 defines a Turing machine \( F \) that decides the language \( EQ_{\text{DFA}} = \{ \langle D, E \rangle \mid D \text{ and } E \text{ are DFAs with } L(D) = L(E) \} \). Then the following Turing machine \( T \) decides \( EQ_{\text{REX}} \):

\[
T = \text{"On input } \langle R, S \rangle \text{, where } R \text{ and } S \text{ are regular expressions:}\n\begin{align*}
1. \ & \text{Convert } R \text{ into an equivalent DFA } D \text{ using Kleene’s Theorem.} \\
2. \ & \text{Convert } S \text{ into an equivalent DFA } E \text{ using Kleene’s Theorem.} \\
3. \ & \text{Run TM } F \text{ from Theorem 4.4 on input } \langle D, E \rangle. \\
4. \ & \text{If } F \text{ accepts, } \text{accept}. \text{ If } F \text{ rejects, } \text{reject}. 
\end{align*}
\]

6. This is Theorem 5.2, whose proof is given on slide 5-11. Specifically, suppose that \( E_{\text{TM}} \) is decidable by some TM \( R \). Then define another TM \( S \) to decide \( A_{\text{TM}} \) as follows:

\[
S = \text{"On input } \langle M, w \rangle \text{, where } M \text{ is a TM and } w \text{ is a string:}\n\begin{align*}
1. \ & \text{Construct a TM } M_1 \text{ from } M \text{ and } w \text{ as follows:} \\
\ & M_1 = \text{"On input } x:\n\ & \quad 1’. \text{ If } x \neq w, \text{ reject.} \\
\ & \quad 2’. \text{ If } x = w, \text{ run } M \text{ on input } w, \text{ and accept if } M \text{ accepts.”} \\
2. \ & \text{Run TM } R \text{ on input } \langle M_1 \rangle. \\
3. \ & \text{If } R \text{ rejects, } \text{accept}; \text{ if } R \text{ accepts, } \text{reject.”} 
\end{align*}
\]

Note that if \( M \) accepts \( w \), then \( L(M_1) = \{w\} \); if \( M \) does not accept \( w \), then \( L(M_1) = \emptyset \). Thus, \( \langle M, w \rangle \in A_{\text{TM}} \) if and only if \( \langle M_1 \rangle \notin E_{\text{TM}} \), so \( S \) decides \( A_{\text{TM}} \). Thus, we have reduced \( A_{\text{TM}} \) to \( E_{\text{TM}} \). But since \( A_{\text{TM}} \) is undecidable, we must also have that \( E_{\text{TM}} \) is undecidable.