Midterm Exam 2
CS 341: Foundations of Computer Science II — **Spring 2012, day section**
Prof. Marvin K. Nakayama

Print family (or last) name: ____

Print given (or first) name: ____

I have read and understand all of the instructions below, and I will obey the Academic Honor Code.

Signature and Date: ____

- This exam has 8 pages in total, numbered 1 to 8. Make sure your exam has all the pages.
- This exam will be 1 hour and 25 minutes in length.
- This is a closed-book, closed-note exam.
- For all problems, follow these instructions:
  1. Give only your answers in the spaces provided. I will only grade what you put in the answer space, and I will take off points for any scratch work in the answer space. Use the scratch-work area or the backs of the sheets to work out your answers before filling in the answer space.
  2. DFA stands for deterministic finite automaton; NFA stands for nondeterministic finite automaton; CFG stands for context-free grammar; PDA stands for pushdown automaton.
  3. For any proofs, be sure to provide a step-by-step argument, with justifications for every step. Unless you are specifically asked to prove a theorem from the book, you may assume that the theorems in the textbook hold; i.e., you do not have to reprove the theorems in the textbook. When using a theorem from the textbook, make sure you provide enough detail so that it is clear which result you are using; e.g., say something like, "By the theorem that states $S^{**} = S^*$, it follows that ..."

Problem	1	2	3	4	5	6	Total
Points							

1. **[20 points]** For each of the following, circle TRUE if the statement is correct. Otherwise, circle FALSE

(a) TRUE FALSE — If A is context-free, then A is Turing-recognizable.

(b) TRUE FALSE — If A is context-free, then A is Turing-decidable.

(c) TRUE FALSE — Every language is Turing-recognizable.

(d) TRUE FALSE — For a Turing machine M and a string w, M either accepts or rejects w.

(e) TRUE FALSE — The language $(0 \cup 1)^*$ is countable.

(f) TRUE FALSE — The set $\mathcal{B}$ of infinite binary sequences is countable.

(g) TRUE FALSE — If language A is recognized by a 14-tape nondeterministic Turing machine, then there is a single-tape deterministic Turing machine that also recognizes A.

(h) TRUE FALSE — The language EQ_{TM} is decidable, where

$$EQ_{\text{TM}} = \{ \langle M, N \rangle \mid M \text{ and } N \text{ are TMs with } L(M) = L(N) \}.$$

(i) TRUE FALSE — The language EQ_{DFA} is decidable, where

$$EQ_{\text{DFA}} = \{ \langle C, D \rangle \mid C \text{ and } D \text{ are DFAs with } L(C) = L(D) \}.$$

(j) TRUE FALSE — The language $\overline{A_{\text{TM}}}$ is decidable, where

$$A_{\text{TM}} = \{ \langle M, w \rangle \mid M \text{ is a TM that accepts input } w \}.$$

2. **[20 points]** Give a short answer (at most three sentences) for each part below. For parts (a), (b) and (c), let $A = \{x, y, z\}$ and $B = \{1, 2, 3, 4\}$, and define the function $f : A \to B$ such that

$$\begin{aligned} f(x) &= 3, \\ f(y) &= 1, \\ f(z) &= 2. \end{aligned}$$

Explain your answers.

(a) Is f one-to-one?

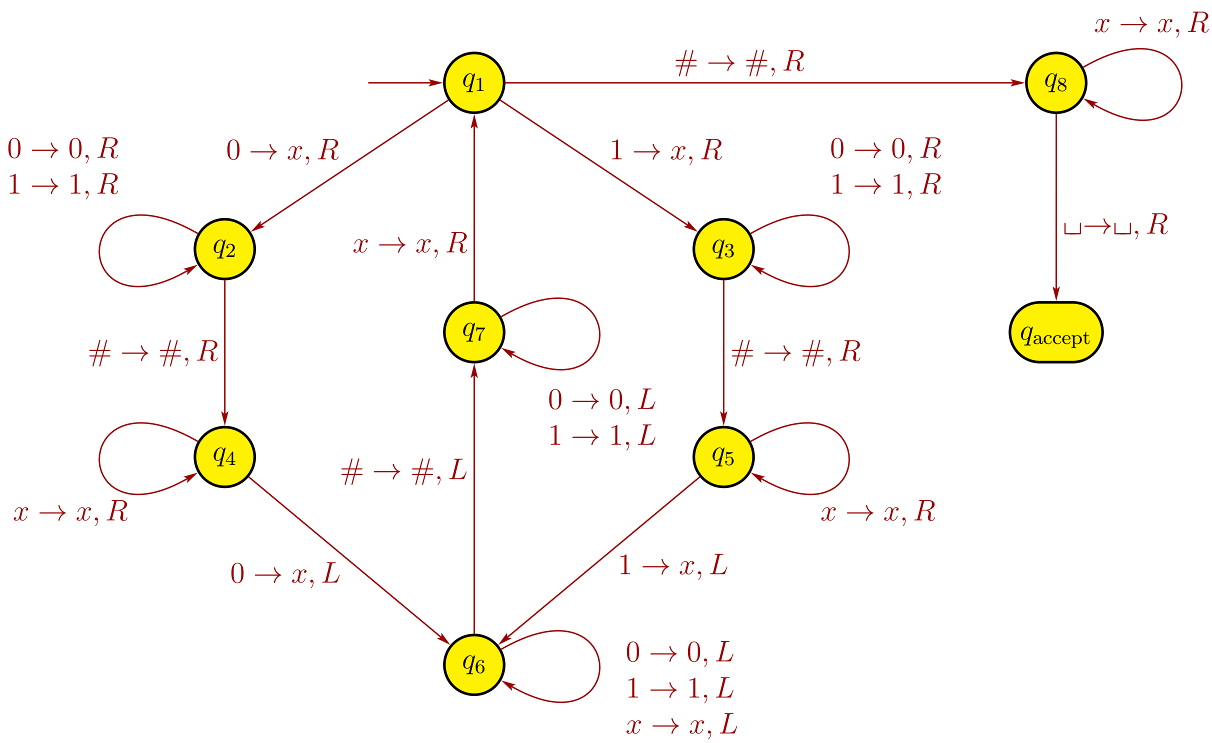
(b) Is f onto?

(c) Is f a correspondence?

(d) What is the difference between a Turing-recognizable language and a Turing-decidable language?

(e) What does the Church-Turing Thesis say?

3. **[20 points]** Consider the below Turing machine $M = (Q, \Sigma, \Gamma, \delta, q_1, q_{\text{accept}}, q_{\text{reject}})$ with $Q = \{q_1, \ldots, q_8, q_{\text{accept}}, q_{\text{reject}}\}$, $\Sigma = \{0, 1, \#\}$, $\Gamma = \{0, 1, \#, x, \sqcup\}$, and transitions below.



To simplify the figure, we don't show the reject state q_{reject} or the transitions going to the reject state. Those transitions occur implicitly whenever a state lacks an outgoing transition for a particular symbol. For example, because in state q_5 no outgoing arrow with a # is present, if a # occurs under the head when the machine is in state q_5, it goes to state q_{reject}. For completeness, we say that in each of these transitions to the reject state, the head writes the same symbol as is read and moves right.

In each of the parts below, give the sequence of configurations that M enters when started on the indicated input string.

(a) 10#11

(b) 0#0

Each of the following problems requires you to prove a result. If you are asked to prove a result A and your proof relies on another result B, then you do not need to prove B if B is a result that we either went over in class or was in the homework. In this case, you need to make clear what result B you are citing in your proof of A (e.g., say something like, "By the result that $S^{**} = S^{*}$ for any set S of strings, we can show that ...").

4. **[10 points]** Let $\mathcal{L}$ be the set of all languages over an alphabet Σ. Prove $\mathcal{L}$ is uncountable.

5. **[15 points]** Consider the problem of determining **if two regular expressions are equivalent**. Express this problem as a language and show that it is decidable.

6. **[15 points]** Recall that

$$E_{\mathrm{TM}} = \{\, \langle M \rangle \mid M \text{ is a Turing machine with } L(M) = \emptyset \,\}.$$

Prove that E_{TM} is undecidable by showing that A_{TM} reduces to E_{TM}, where

$$A_{\mathrm{TM}} = \{\, \langle M, w \rangle \mid M \text{ is a Turing machine that accepts input } w \,\}.$$