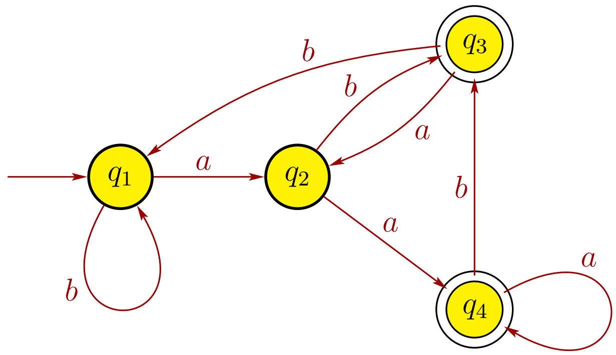
CS 341, Fall 2013
Solutions for Midterm, eLearning Section

1. (a) True. Since A is finite, it is regular by slide 1-81. Thus, $\overline{A}$ is regular by Homework 2, problem 3. Also, B is regular since it has a regular expression (Theorem 1.54), so $\overline{A} \cap B$ is regular by slide 1-28. Hence, Corollary 2.32 implies $\overline{A} \cap B$ is context-free.
   (b) False. The derivation $S \Rightarrow 0$ generates the string 0, which is not in the language, so the CFG cannot be correct.
   (c) True. Homework 5, problem 3b.
   (d) True. Since A has a PDA, it is context-free by Theorem 2.20, so the statement then follows from Theorem 2.9.
   (e) True. The language $\emptyset$ is finite, so slide 1-81 shows that it is regular. Corollary 2.32 then implies that $\emptyset$ is also context-free.
   (f) False. For example, let A have regular expression $(0 \cup 1)^*$, so it is an infinite language. Since A has a regular expression, it is a regular language by Theorem 1.54.
   (g) True. By Corollary 1.40, A is regular since it has an NFA. Corollary 2.32 then implies that A is context-free, so it has a PDA by Theorem 2.20.
   (h) False. The language $A = \{ a^n b^n c^n \mid n \geq 0 \}$ is nonregular, which we can show by the pumping lemma for regular languages (choose the string $s = a^p b^p c^p \in A$ to get a contradiction). But slide 2-105 shows that A is also non-context-free.
   (i) False. Let $A = \{ 0^n 1^n \mid n \geq 0 \}$ and B have regular expression $(0 \cup 1)^*$. Then B is regular since it has a regular expression (Theorem 1.54). Also, note that $A \subseteq B$, but A is nonregular, as shown on slide 1-90.
   (j) False. Homework 6, problem 2b.

2. (a) $b^*(a \cup ab^*(a \cup \varepsilon))b^*$. There are infinitely many correct regular expressions.
   (b)
   - $S \to Y$ is not in Chomsky normal form since unit rules are not allowed.
   - $S \to ba$ is improper since a rule cannot have more than one terminal on the RHS.
   - $X \to XS$ is improper since starting variable S cannot be on RHS of rule.
   - $X \to \varepsilon$ is improper since ε cannot be on the RHS of rule when the left side is not S.
   - $Y \to XXY$ is improper since the RHS cannot have more than two variables.

   (c) slide 1-50.
   (d) This is Homework 5, problem 3a. Define CFG $G_3 = (V_3, \Sigma, R_3, S_3)$ with $V_3 = V_1 \cup V_2 \cup \{S_3\}$ and $R_3 = R_1 \cup R_2 \cup \{ S_3 \to S_1,\ S_3 \to S_2 \}$.

3. (a) Below is a DFA for the language A. There are infinitely many other correct DFAs for A.



(b) A regular expression for A is $(a \cup b)^* a(a \cup b)$. There are infinitely many other regular expressions for A.

4. This is Homework 6, problem 2a. Define languages

$$A = \{ a^m b^n c^n \mid m, n \geq 0 \} \text{ and}$$
$$B = \{ a^n b^n c^m \mid m, n \geq 0 \}.$$

The language A is context free since it has CFG G_1 with rules

$$\begin{aligned} S &\rightarrow XY \\ X &\rightarrow aX \mid \varepsilon \\ Y &\rightarrow bYc \mid \varepsilon \end{aligned}$$

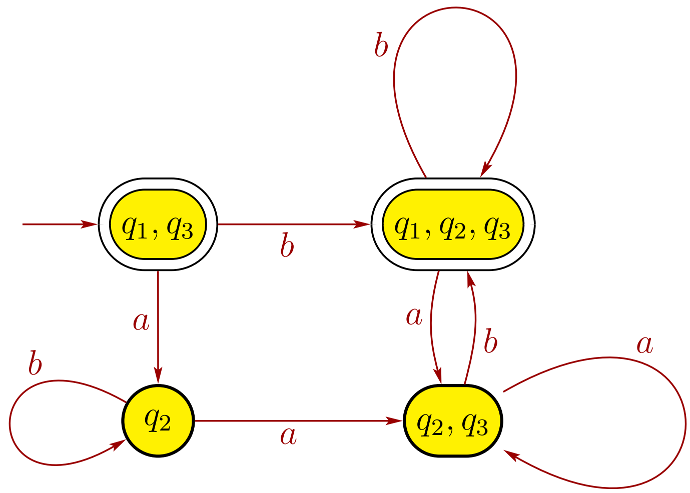
The language B is context free since it has CFG G_2 with rules

$$\begin{aligned} S &\rightarrow XY \\ X &\rightarrow aXb \mid \varepsilon \\ Y &\rightarrow cY \mid \varepsilon \end{aligned}$$

But $A \cap B = \{ a^n b^n c^n \mid n \geq 0 \}$, which we know is not context free from Example 2.36 of the textbook. Thus, the class of context-free languages is not closed under intersection.

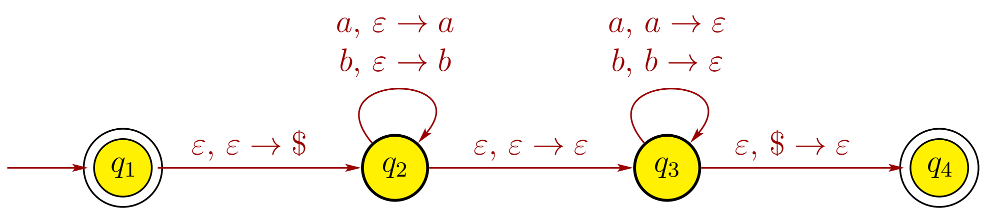
5. (a) $\varepsilon, b, bb, aab, bab, \ldots$

(b)



6. (a) $G = (V, \Sigma, R, S)$, with $V = \{S\}$, $\Sigma = \{a, b\}$, start variable S and rules $S \to aSa \mid bSb \mid \varepsilon$.

   (b) The following PDA is the same as one on slide 2-59 except the one in the notes has alphabet $\Sigma = \{0, 1\}$ instead of $\Sigma = \{a, b\}$.



7. Suppose that A is a regular language. Let p be the pumping length, and consider the string $s = a^p bba^p \in A$. Note that $|s| = 2p + 2 \geq p$, so the pumping lemma implies we can write $s = xyz$ with $xy^iz \in A$ for all $i \geq 0$, $|y| > 0$, and $|xy| \leq p$. Now, $|xy| \leq p$ implies that x and y have only a's (together up to p in total) and z has the rest of the a's at the beginning, followed by bba^p. Hence, we can write $x = a^j$ for some $j \geq 0$, $y = a^k$ for some $k \geq 0$, and $z = a^\ell bba^p$, where $j + k + \ell = p$ since $xyz = s = a^p bba^p$. Also, $|y| > 0$ implies $k > 0$. Now consider the string $xyyz = a^j a^k a^k a^\ell bba^p = a^{p+k} bba^p$ since $j + k + \ell = p$. Note that $xyyz \notin A$ since $k \geq 1$ so the string is not the same forwards as backwards. This contradicts (i), so A is not a regular language.