Midterm Exam
CS 341-451: Foundations of Computer Science II — **Fall 2013, eLearning section**
Prof. Marvin K. Nakayama

Print family (or last) name: ____

Print given (or first) name: ____

I have read and understand all of the instructions below, and I will obey the Academic Honor Code.

____

Signature and Date

- This exam has 9 pages in total, numbered 1 to 9. Make sure your exam has all the pages.
- Unless other arrangements have been made with the professor, the exam is to last 2.5 hours and is to be given on Saturday, October 16, 2010.
- This is a closed-book, closed-note exam. No calculators are allowed.
- For all problems, follow these instructions:
  1. Give only your answers in the spaces provided. I will only grade what you put in the answer space, and I will take off points for any scratch work in the answer space. Use the scratch-work area or the backs of the sheets to work out your answers before filling in the answer space.
  2. DFA stands for deterministic finite automaton; NFA stands for nondeterministic finite automaton; CFG stands for context-free grammar; PDA stands for pushdown automaton.
  3. For any proofs, be sure to provide a step-by-step argument, with justifications for every step. If you are asked to prove a result X, in your proof of X, you may use any other result Y without proving Y. However, make it clear what the other result Y is that you are using; e.g., write something like, "By the result that $A^{**} = A^*$, we know that ...."

Problem	1	2	3	4	5	6	7	Total
Points								

1. **[20 points]** For each of the following, circle TRUE if the statement is correct. Otherwise, circle FALSE

(a) TRUE FALSE — If A is a finite language and B has a regular expression, then $\overline{A} \cap B$ must be context-free.

(b) TRUE FALSE — The language $\{ 1^n 0^n \mid n \geq 0 \}$ has context-free grammar $G = (V, \Sigma, R, S)$, with $V = \{S\}$, $\Sigma = \{0, 1\}$, start variable S, and rules $S \to 1S0 \mid 0$.

(c) TRUE FALSE — The class of context-free languages is closed under concatenation.

(d) TRUE FALSE — If A has a PDA, then A must have a CFG in Chomsky normal form.

(e) TRUE FALSE — $\emptyset$ is a context-free language.

(f) TRUE FALSE — If A is a regular language, then A is finite.

(g) TRUE FALSE — If A has an NFA, then A must have a PDA.

(h) TRUE FALSE — Every nonregular language is context-free.

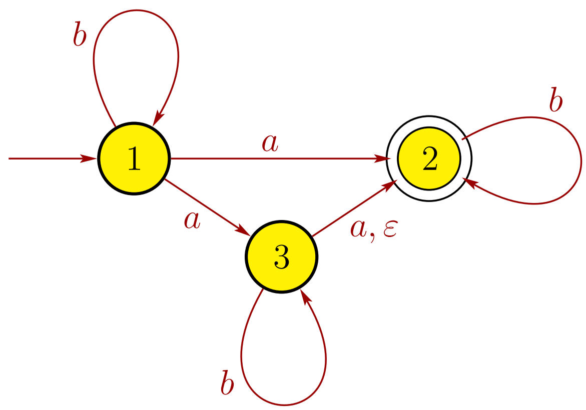
(i) TRUE FALSE — If A and B are languages such that $A \subseteq B$ and B is regular, then A must be regular.

(j) TRUE FALSE — The class of context-free languages is closed under complements.

2. **[20 points]** Give short answers to each of the following parts. **Each answer should be at most a few sentences. Be sure to define any notation that you use.**

(a) Give a regular expression for the language recognized by the NFA below.

Answer: ____



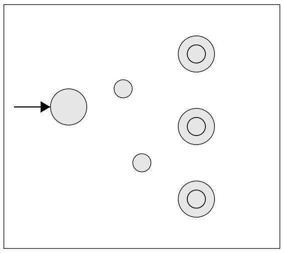
(b) Consider the following CFG $G = (V, \Sigma, R, S)$, with $V = \{S, X, Y\}$, $\Sigma = \{a, b\}$, start variable S, and rules R as follows:

$$\begin{aligned} S &\rightarrow Y \mid XX \mid ba \mid \varepsilon \\ X &\rightarrow XS \mid \varepsilon \\ Y &\rightarrow a \mid XXY \end{aligned}$$

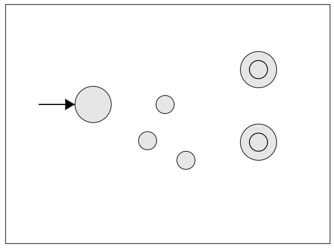
Note that G is not in Chomsky normal form. List all of the rules in G that violate Chomsky normal form. Explain your answer.

(c) Suppose that language A_1 is recognized by NFA N_1 below, and language A_2 is recognized by NFA N_2 below. Note that the transitions are not drawn in N_1 and N_2. Draw a picture of an NFA for $A_1 \circ A_2$.

N_1



N_2



(d) Suppose that A_1 is a language defined by a CFG $G_1 = (V_1, \Sigma, R_1, S_1)$, and A_2 is a language defined by a CFG $G_2 = (V_2, \Sigma, R_2, S_2)$, where the alphabet Σ is the same for both languages and $V_1 \cap V_2 = \emptyset$. Let $A_3 = A_1 \cup A_2$. Give a CFG G_3 for A_3 in terms of G_1 and G_2. You do not have to prove the correctness of your CFG G_3, but do not give just an example.

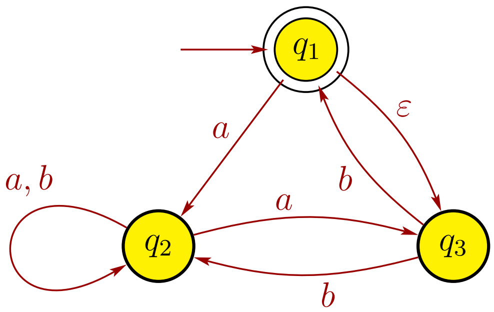
3. [**10 points**] Let $\Sigma = \{a, b\}$, and define $A = \{\, w \in \Sigma^* \mid \text{the second-to-last symbol in } w \text{ is } a \,\}$. If string $w = w_1 w_2 \cdots w_n$ where each $w_i \in \Sigma$, then the second-to-last symbol of w is w_{n-1}.

(a) Give a DFA for A. Only give the drawing of the DFA; do not give its 5-tuple definition.

(b) Give a regular expression for A.

4. [**10 points**] Give an example of context-free languages A and B such that $C = A \cap B$ is not context-free. Explain your answer. Be sure to give CFGs for A and B. You do not have to prove that C is non-context-free for your example, but C must be a non-context-free language that we went over in the course.

5. **[15 points]** Let N be the following NFA with $\Sigma = \{a, b\}$, and let $C = L(N)$.



(a) List the strings in C in lexicographic order. If C has more than 5 strings, list only the first 5 strings in C, followed by 3 dots.

(b) Give a DFA for C.

Scratch-work area

6. **[15 points]** Let $\Sigma = \{a, b\}$, and consider the language $A = \{ w \in \Sigma^* \mid w = w^{\mathcal{R}}, |w| \text{ is even} \}$, where $w^{\mathcal{R}}$ denotes the reverse of w and $|w|$ denotes the length of w.

(a) Give a CFG G for A. Be sure to specify G as a 4-tuple $G = (V, \Sigma, R, S)$.

(b) Give a PDA for A. You only need to give the drawing.

Scratch-work area

7. **[10 points]** Recall the pumping lemma for regular languages:

Theorem: If L is a regular language, then there is a number p (pumping length) where, if $s \in L$ with $|s| \geq p$, then there are strings x, y, z such that $s = xyz$ and

(i) $xy^iz \in L$ for each $i \geq 0$,

(ii) $|y| > 0$, and

(iii) $|xy| \leq p$.

Let $\Sigma = \{a, b\}$, and consider the language $A = \{\, w \in \Sigma^* \mid w = w^{\mathcal{R}}, |w| \text{ is even} \,\}$, where $w^{\mathcal{R}}$ denotes the reverse of w and $|w|$ denotes the length of w. Prove that A is not a regular language.