Midterm Exam 1
CS 341: Foundations of Computer Science II — **Fall 2013, day section**
Prof. Marvin K. Nakayama

Print family (or last) name: ____________________________________

Print given (or first) name: ____________________________________

I have read and understand all of the instructions below, and I will obey the Academic Honor Code.

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Signature and Date

- This exam has 9 pages in total, numbered 1 to 9. Make sure your exam has all the pages.
- Note the number written on the upper right-hand corner of the first page. On the sign-up sheet being passed around, sign your name next to this number.
- This exam will be 1 hour and 25 minutes in length.
- This is a closed-book, closed-note exam.
- For all problems, follow these instructions:
  1. Give only your answers in the spaces provided. I will only grade what you put in the answer space, and I will take off points for any scratch work in the answer space. Use the scratch-work area or the backs of the sheets to work out your answers before filling in the answer space.
  2. For any proofs, be sure to provide a step-by-step argument, with justifications for every step.

Problem	1	2	3	4	5	Total
Points						

1. **[20 points]** For each of the following, circle TRUE if the statement is correct. Otherwise, circle FALSE

(a) TRUE FALSE — If A and B are regular languages, then $(\overline{A} \cup B)^*$ is regular.

(b) TRUE FALSE — The class of context-free languages is closed under intersection.

(c) TRUE FALSE — If B is a context-free language and $A \subseteq B$, then A is context-free.

(d) TRUE FALSE — The language $\{ a^n b^n \mid n \geq 3 \}$ is non-regular.

(e) TRUE FALSE — If a language A has a regular expression, then A has a CFG in Chomsky normal form.

(f) TRUE FALSE — If A is a non-context-free language, then A is also non-regular.

(g) TRUE FALSE — The class of context-free languages is closed under union.

(h) TRUE FALSE — If a finite number of strings is added to a regular language A, then the resulting language is regular.

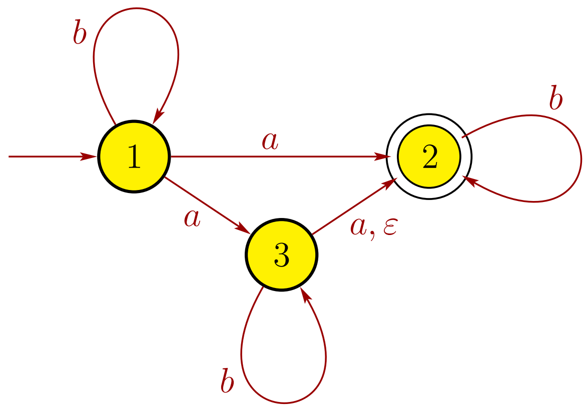
(i) TRUE FALSE — If a finite number of strings is added to a nonregular language A, then the resulting language is nonregular.

(j) TRUE FALSE — If A is a regular language, then A is finite.

2. **[20 points]** Give short answers to each of the following parts. Be sure to define any notation that you use.

(a) Give a regular expression for the language recognized by the NFA below.

Answer: ______________________________



(b) Suppose A is generated by a context-free grammar $G = (V, \Sigma, R, S)$. Give a context-free grammar G' for A^* in terms of G. You do not have to prove the correctness of your CFG G', but do not just give an example.

(c) Let $M_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$ be a DFA with language A_1, and $M_2 = (Q_2, \Sigma, \delta_2, q_2, F_2)$ be a DFA with language A_2. Consider the language $A = A_1 \cap A_2$. Give a DFA M_3 for A in terms of M_1 and M_2. Your DFA M_3 must be completely general. Do not prove the correctness of your DFA M_3, but do not just give an example.

(d) Suppose that we are in the process of converting a CFG G with $\Sigma = \{0, 1\}$ into Chomsky normal form. We have already applied some steps in the process, and we currently have the following CFG:

$$\begin{aligned} S_0 &\rightarrow S \\ S &\rightarrow A1SA \mid A0 \mid \varepsilon \\ A &\rightarrow 0S0 \mid \varepsilon \end{aligned}$$

In the next step, we want to remove the ε-rule $A \rightarrow \varepsilon$. Give the CFG after carrying out just this one step.

3. **[25 points]** Define L to be the set of strings that represent numbers in a modified version of Java. The goal in this problem is to define a regular expression and an NFA for L. To precisely define L, let $\Sigma = \Sigma_1 \cup \Sigma_2 \cup \{.\}$, where $\Sigma_1 = \{\texttt{0}, \texttt{1}, \texttt{2}, \ldots, \texttt{9}\}$ is the set of *digits* and $\Sigma_2 = \{\texttt{+}, \texttt{-}\}$ is the set of *signs*. Then $L = L_1 \cup L_2$, where

- L_1 is the set of all strings that are decimal integer numbers. Specifically, L_1 consists of strings that start with an optional sign, followed by one or more digits. Examples of strings in L_1 are "`02`", "`+9`", and "`-241`".
- L_2 is the set of all strings that are floating-point numbers that are not in exponential notation. Specifically, L_2 consists of strings that start with an optional sign, followed by zero or more digits, followed by a decimal point, and end with zero or more digits, where there must be at least one digit in the string. Examples of strings in L_2 are "`13.231`", "`-28.`" and "`.124`". All strings in L_2 have exactly one decimal point.

Assume that there is no limit on the number of digits in a string in L. Also, we do not allow exponential notation, nor do we allow for the suffixes `L`, `l`, `F`, `f`, `D`, `d`, at the end of numbers to denote types (long integers, floats, and doubles); these symbols are not in Σ anyways.

(a) Give a regular expression for L_1.

(b) Give an NFA for L_1 over the alphabet Σ.

(c) Give a DFA for L_1 over the alphabet Σ. Your DFA must include all transitions.

(d) Give a regular expression for L_2.

(e) Give an NFA for L_2 over the alphabet Σ.

(f) Give a regular expression for L.

(g) Give an NFA for L over the alphabet Σ.

4. **[20 points]** Consider the alphabet $\Sigma = \{a, b, c\}$ and the language

$$L = \{\ b^i a^j c^k \mid i, j, k \geq 0 \text{ and } i + j = k\ \}.$$

(a) Give a context-free grammar G for L. Be sure to specify G as a 4-tuple $G = (V, \Sigma, R, S)$.

(b) Give a PDA for L. You only need to draw the graph.

Scratch-work area

5. **[15 points]** Recall the pumping lemma for regular languages:

Theorem: If L is a regular language, then there exists a pumping length p where, if $s \in L$ with $|s| \geq p$, then there exists strings x, y, z such that $s = xyz$ and (i) $xy^iz \in L$ for each $i \geq 0$, (ii) $|y| \geq 1$, and (iii) $|xy| \leq p$.

Let $A = \{ w \in \{a, b\}^* \mid w \text{ has more } a\text{'s than } b\text{'s} \}$. Is A a regular or nonregular language? If A is regular, give a regular expression for A. If A is not regular, prove that it is a nonregular language.

Circle one: **Regular Language** **Nonregular Language**