CS 341, Fall 2014
Solutions for Midterm, eLearning Section

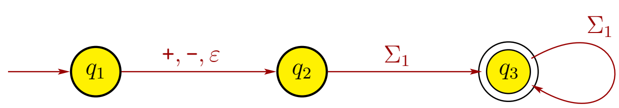
1. (a) True. Theorem 2.20.
   (b) True. If A is finite, then it must be regular (slide 1-95).
   (c) False. $\{0, 1\}^*$ is a regular language that is infinite.
   (d) True, by Kleene's Theorem.
   (e) False. 0^*1^* generates $00111 \notin \{0^n1^n \mid n \geq 0\}$.
   (f) False. $A \times B$ contains pairs of strings, whereas $A \circ B$ contains just strings.
   (g) True. Homework 2, problem 3.
   (h) False. $A = \{0^n1^n \mid n \geq 0\}$ is a context-free language, but it is nonregular. Hence, A cannot have an NFA.
   (i) True. Homework 5, problem 3(b).
   (j) True. Homework 5, problem 3(a).

2. (a) A regular expression for L_1 is

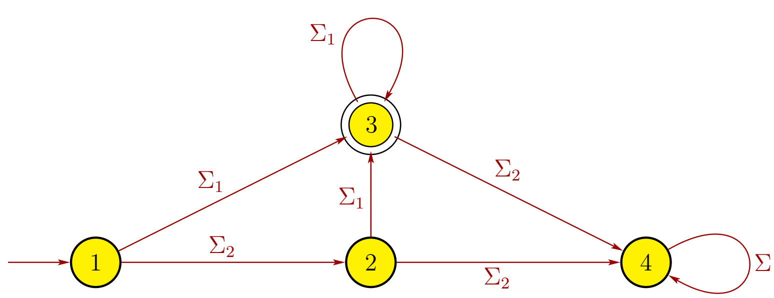
$$R_1 = (\,\texttt{+} \cup \texttt{-} \cup \varepsilon\,)\,\Sigma_1\,\Sigma_1^*$$

   where $\Sigma_1 = \{\,\texttt{0},\,\texttt{1},\,\texttt{2},\,\ldots,\,\texttt{9}\,\}$ as defined in the problem.

   (b) An NFA for L_1 is



   (c) Define $\Sigma_2 = \{\,\texttt{-},\,\texttt{+}\,\}$, as given in the problem. Then a DFA for L_1 is

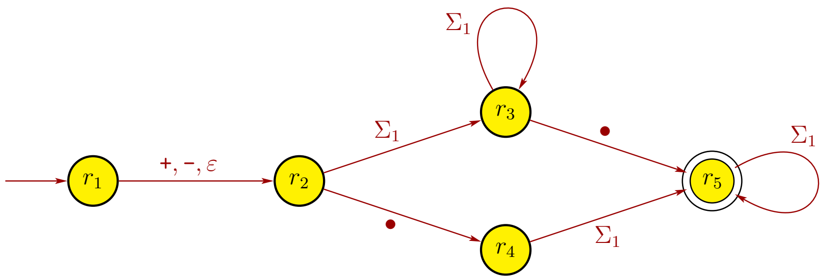


(d) A regular expression for L_2 is

$$R_2 = (\,\texttt{+} \cup \texttt{-} \cup \varepsilon\,)(\Sigma_1\,\Sigma_1^*\,\texttt{.}\,\Sigma_1^* \;\cup\; \texttt{.}\,\Sigma_1\,\Sigma_1^*)$$

Note that the regular expression $(\,\texttt{+} \cup \texttt{-} \cup \varepsilon\,)\,\Sigma_1^*\,\texttt{.}\,\Sigma_1^*$ is not correct since it can generate the strings "`.`", "`+.`" and "`-.`", which are not valid floating-point numbers.

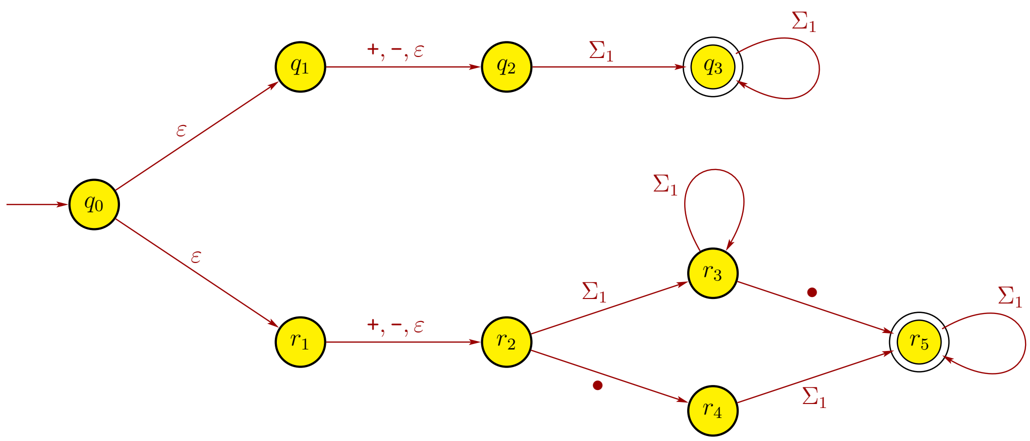
(e) An NFA for L_2 is



(f) Note that $L = L_1 \cup L_2$, so a regular expression for L is

$$R_3 = R_1 \cup R_2$$

(g) We can construct an NFA for L by taking the union of the NFA's for L_1 and L_2 as follows:



There are many other correct answers for this and the other parts.

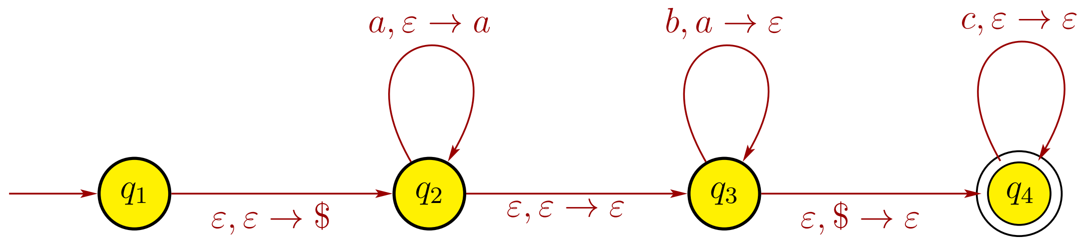
3. (a) $q_1bb \quad \sqcup q_2b \quad \sqcup xq_3\sqcup \quad \sqcup q_5x \quad q_5\sqcup x \quad \sqcup q_2x \quad \sqcup xq_2\sqcup \quad \sqcup x\sqcup q_{\text{accept}}$

   (b) $q_1bbbbb \quad \sqcup q_2bbbb \quad \sqcup xq_3bbb \quad \sqcup xbq_4bb \quad \sqcup xbxq_3b \quad \sqcup xbxbq_4$ $\sqcup xbxb\sqcup q_{\text{reject}}$

4. (a) CFG $G = (V, \Sigma, R, S)$, with $V = \{S, X, Y\}$ and start variable S, $\Sigma = \{a, b, c\}$, and rules R:

$$S \to XY$$
$$X \to aXb \mid \varepsilon$$
$$Y \to cY \mid \varepsilon$$

(b) PDA



There are other correct PDAs that recognize A.

5. This is Homework 6, problem 2a. Define languages

$$\begin{aligned} A &= \{\, a^m b^n c^n \mid m, n \geq 0 \,\} \text{ and} \\ B &= \{\, a^n b^n c^m \mid m, n \geq 0 \,\}. \end{aligned}$$

The language A is context free since it has CFG G_1 with rules

$$\begin{aligned} S &\to XY \\ X &\to aX \mid \varepsilon \\ Y &\to bYc \mid \varepsilon \end{aligned}$$

The language B is context free since it has CFG G_2 with rules

$$\begin{aligned} S &\to XY \\ X &\to aXb \mid \varepsilon \\ Y &\to cY \mid \varepsilon \end{aligned}$$

But $A \cap B = \{\, a^n b^n c^n \mid n \geq 0\}$, which we know is not context free from Example 2.36 of the textbook. Thus, the class of context-free languages is not closed under intersection.

6. Suppose that A is a regular language. Let p be the pumping length, and consider the string $s = a^p b^p \in A$. Note that $|s| = 2p \geq p$, so the pumping lemma implies we can write $s = xyz$ with $xy^iz \in A$ for all $i \geq 0$, $|y| > 0$, and $|xy| \leq p$. Now, $|xy| \leq p$ implies that x and y have only a's (together up to p in total) and z has the rest of the a's at the beginning, followed by b^p. Hence, we can write $x = a^j$ for some $j \geq 0$, $y = a^k$ for some $k \geq 0$, and $z = a^\ell b^p$, where $j + k + \ell = p$ since $xyz = s = a^p b^p$. Also, $|y| > 0$ implies $k > 0$. Now consider the string $xyyz = a^j a^k a^k a^\ell b^p = a^{p+k} b^p$ since $j + k + \ell = p$. Note that $xyyz \notin A$ since $k > 0$ so the number of a's and b's are not equal. This contradicts (i), so A is not a regular language.