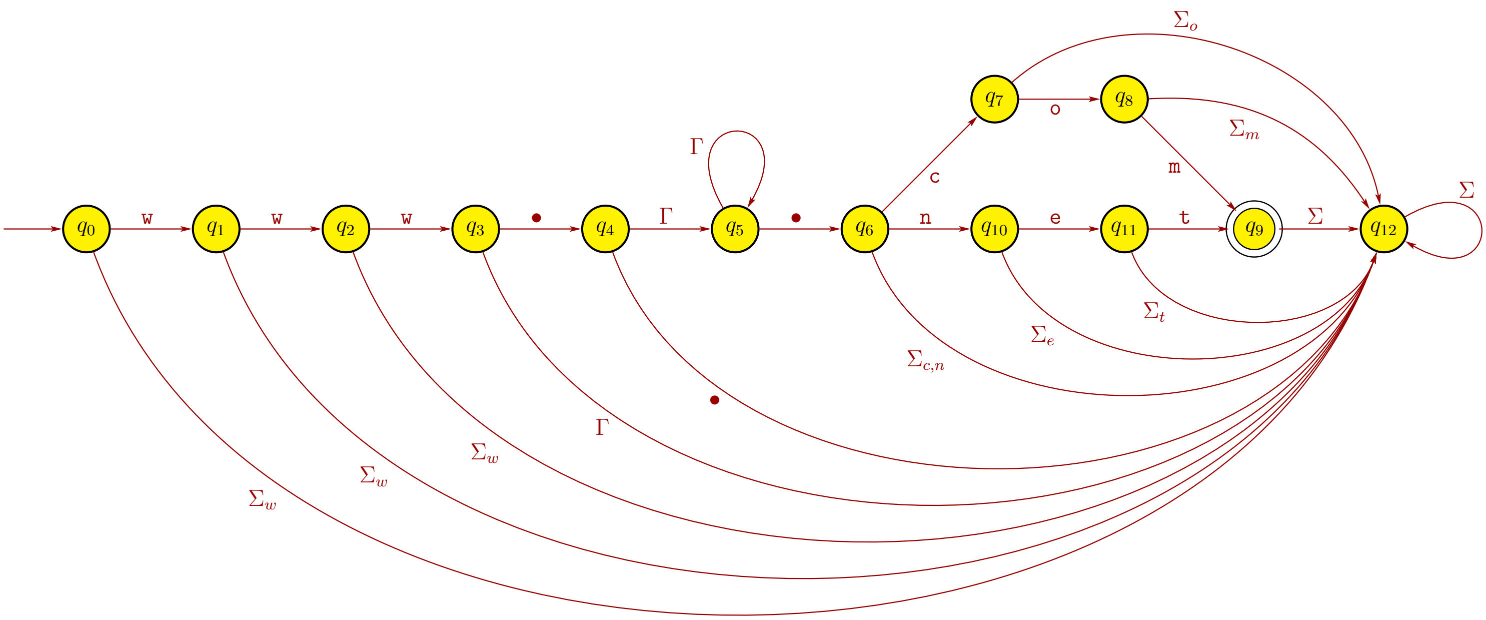
CS 341, Spring 2014
Solutions for Midterm, eLearning Section

1. (a) False. Theorem 1.39.
   (b) True. By Theorem 2.9. The fact that A is non-regular is irrelevant.
   (c) True. Suppose A is non-context-free but regular. But then Corollary 2.32 implies A is context-free, which is a contradiction.
   (d) False. The language a^* is regular but infinite.
   (e) False. The TM M can also loop on w.
   (f) False. $\{a^n b^n c^n \mid n \geq 0\}$ is non-regular, but not context-free.
   (g) False. HW 6, problem 2(a).
   (h) True. Kleene's Theorem ensures A^* is regular, and we know $\overline{B}$ is regular by HW 2, problem 3. Thus, $A^* \cap \overline{B}$ is regular by HW 2, problem 5.
   (i) True. Corollary 2.32 implies A is context-free. Thus, A has a PDA by Theorem 2.20.
   (j) False. For example, $A = \emptyset$ is a subset of $B = \{0^n 1^n \mid n \geq 0\}$, but A is regular and B is non-regular.

2. (a) $\varepsilon \cup a \cup b \cup (a \cup b)^*(ab \cup ba)$
   (b)
   - $S \to X$ is improper since it is a unit rule.
   - $S \to ba$ is improper since a rule can't have more than one terminal on the right side.
   - $X \to YS$ is improper since the start variable S can't be on the right side.
   - $X \to \varepsilon$ is improper if ε is on the right side, S must be on the left side.
   - $Y \to aX$ is improper since the right side has a mix of terminals and variables.

   (c) slide 1-53.
   (d) Homework 5, problem 3a.

3. Below is a DFA for the language L. There are other correct DFAs for L.



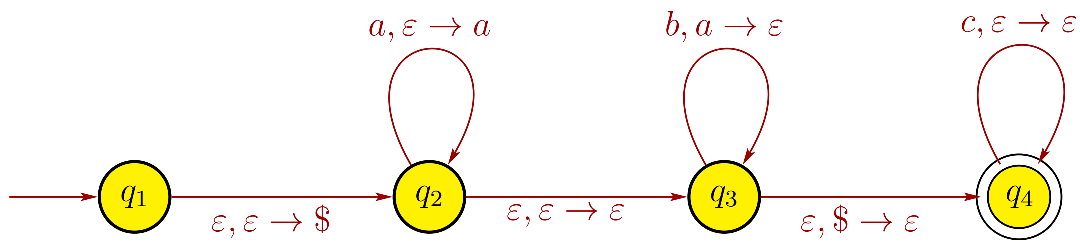
4. (a) $q_1 110\#01 \quad xq_3 10\#01 \quad x1q_3 0\#01 \quad x10q_3\#01 \quad x10\#q_5 01 \quad x10\#0q_{\text{reject}}1$

   (b) $q_1 0\#0 \quad xq_2\#0 \quad x\#q_4 0 \quad xq_6\#x \quad q_7 x\#x \quad xq_1\#x \quad x\#q_8 x \quad x\#xq_8$
   $x\#x \sqcup q_{\text{accept}}$

5. (a) CFG $G = (V, \Sigma, R, S)$, with $V = \{S, X, Y\}$ and start variable S, $\Sigma = \{a, b, c\}$, and rules R:

$$S \to XY$$
$$X \to aXb \mid \varepsilon$$
$$Y \to cY \mid \varepsilon$$

   (b) PDA



6. HW 6, problem 2b.

7. Suppose that A is a regular language. Let p be the pumping length, and consider the string $s = a^p b^p \in A$. Note that $|s| = 2p \geq p$, so the pumping lemma implies we can write $s = xyz$ with $xy^i z \in A$ for all $i \geq 0$, $|y| > 0$, and $|xy| \leq p$. Now, $|xy| \leq p$ implies that x and y have only a's (together up to p in total) and z has the rest of the a's at the beginning, followed by b^p. Hence, we can write $x = a^j$ for some $j \geq 0$, $y = a^k$ for some $k \geq 0$, and $z = a^\ell b^p$, where $j + k + \ell = p$ since $xyz = s = a^p b^p$. Also, $|y| > 0$ implies $k > 0$. Now consider the string $xyyz = a^j a^k a^k a^\ell b^p = a^{p+k} b^p$ since $j + k + \ell = p$. Note that $xyyz \notin A$ since $k > 0$ so the number of a's and b's are not equatl. This contradicts (i), so A is not a regular language.