CS 341, Fall 2015
Solutions for Midterm, eLearning Section

1. (a) False. By Homework 6, problem 2(b).
   (b) False. $A = \{a^n b^n c^n \mid n \geq 0\}$ is nonregular and not context-free.
   (c) False. The language a^* is regular but infinite.
   (d) True. Since A is finite, it is regular by slide 1-95 of the notes. Corollary 2.32 then ensures that A is regular.
   (e) False. For example, let $A = \{a^n b^n \mid n \geq 0\}$ and $B = (a \cup b)^*$. Then $A \subseteq B$, B is regular, but A is nonregular.
   (f) False. $a^* b^*$ generates the string $abb \notin \{a^n b^n \mid n \geq 0\}$.
   (g) True. By Theorem 2.9. The fact that A is non-regular is irrelevant.
   (h) True. Since A has an NFA, it is regular by Corollary 1.40. Since B is finite, it is regular by slide 1-95, and we know $\overline{B}$ is regular by HW 2, problem 3. Thus, $A \cap \overline{B}$ is regular by HW 2, problem 5, so Corollary 2.32 implies $A \cap \overline{B}$ is context-free.
   (i) False. A TM can loop on w.
   (j) False. For example, let $A = \{abc\}$ and $B = \{a^n b^n c^n \mid n \geq 0\}$, so $A \subseteq B$. Because A is finite, it is regular (slide 1-95), so it is also context-free by Corollary 2.32. But B is not context-free by slide 2-96.

2. (a) $\varepsilon \cup a \cup b \cup (a \cup b)^*(ab \cup ba)$
   (b) After one step, the CFG is then

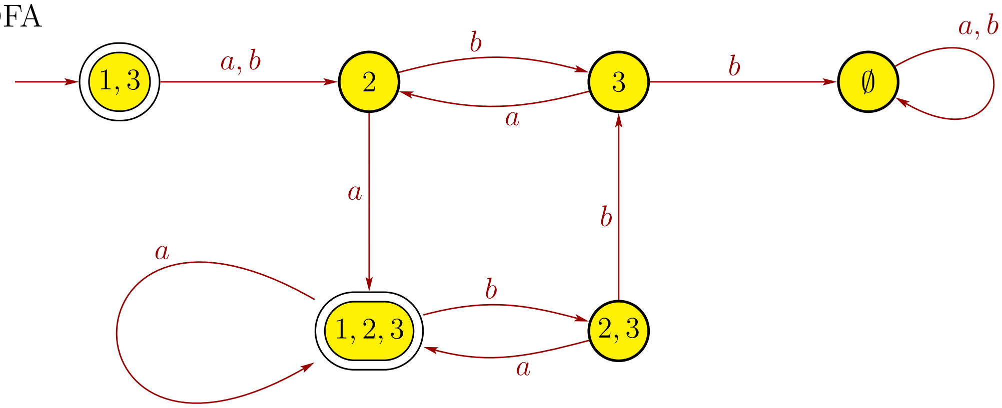
$$\begin{aligned} S_0 &\rightarrow S \\ S &\rightarrow A1SA \mid 1SA \mid A1S \mid 1S \mid A0 \mid 0 \mid \varepsilon \\ A &\rightarrow 0S0 \end{aligned}$$

   (c) slide 1-66.
   (d) Homework 5, problem 3a.

3. $q_1bb \quad \sqcup q_2b \quad \sqcup x q_3 \sqcup \quad \sqcup q_5 x \quad q_5 \sqcup x \quad \sqcup q_2 x \quad \sqcup x q_2 \sqcup \quad \sqcup x \sqcup q_{\text{accept}}$

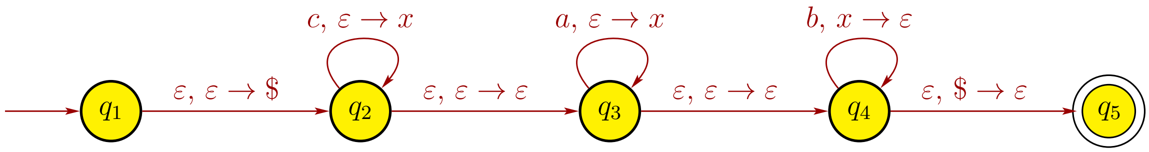
4. Homework 3, problem 2.

5. DFA



6. (a) $G = (V, \Sigma, R, S)$, with $V = \{S, X\}$, $\Sigma = \{a, b, c\}$, start variable S and rules $S \to cSb \mid X$ and $X \to aXb \mid \varepsilon$.

   (b) PDA



   For every c and a read in the first part of the string, the PDA pushes an x onto the stack. Then it must read a b for each x popped off the stack.

7. Suppose that A is a regular language. Let p be the pumping length, and consider the string $s = a^p bba^p \in A$. Note that $|s| = 2p + 2 \geq p$, so the pumping lemma implies we can write $s = xyz$ with $xy^iz \in A$ for all $i \geq 0$, $|y| > 0$, and $|xy| \leq p$. Now, $|xy| \leq p$ implies that x and y have only a's (together up to p in total) and z has the rest of the a's at the beginning, followed by bba^p. Hence, we can write $x = a^j$ for some $j \geq 0$, $y = a^k$ for some $k \geq 0$, and $z = a^\ell bba^p$, where $j + k + \ell = p$ since $xyz = s = a^p bba^p$. Also, $|y| > 0$ implies $k > 0$. Now consider the string $xyyz = a^j a^k a^k a^\ell bba^p = a^{p+k} bba^p$ since $j + k + \ell = p$. Note that $xyyz \notin A$ since it is not the same forwards and backwards because $k > 0$, which contradicts (i), so A is not a regular language.