Midterm Exam 1
CS 341: Foundations of Computer Science II — **Fall 2015, day section**
Prof. Marvin K. Nakayama

Print family (or last) name: ____

Print given (or first) name: ____

I have read and understand all of the instructions below, and I will obey the University Code on Academic Integrity.

____
Signature and Date

- This exam has 7 pages in total, numbered 1 to 7. Make sure your exam has all the pages.
- Note the number written on the upper right-hand corner of the first page. On the sign-up sheet being passed around, sign your name next to this number.
- This exam will be 1 hour and 25 minutes in length.
- This is a closed-book, closed-note exam. Electronic devices (e.g., cellphone, smart watch, calculator) are not allowed.
- For all problems, follow these instructions:
    1. Give only your answers in the spaces provided. I will only grade what you put in the answer space, and I will take off points for any scratch work in the answer space. Use the scratch-work area or the backs of the sheets to work out your answers before filling in the answer space.
    2. For any proofs, be sure to provide a step-by-step argument, with justifications for every step.

Problem	1	2	3	4	5	Total
Points						

1. **[20 points]** For each of the following, circle TRUE if the statement is correct. Otherwise, circle FALSE

(a) TRUE FALSE — If A is a regular language, then A is finite.

(b) TRUE FALSE — If A and B are regular languages, then $(\overline{A} \cup B)^*$ is regular.

(c) TRUE FALSE — The class of context-free languages is closed under intersection.

(d) TRUE FALSE — If B is a context-free language and $A \subseteq B$, then A is context-free.

(e) TRUE FALSE — The language $\{ a^n b^n \mid n \geq 3\}$ is non-regular.

(f) TRUE FALSE — If a language A has a regular expression, then A has a CFG in Chomsky normal form.

(g) TRUE FALSE — The class of context-free languages is closed under union.

(h) TRUE FALSE — If a finite number of strings is added to a regular language A, then the resulting language is regular.

(i) TRUE FALSE — If a finite number of strings is added to a nonregular language A, then the resulting language is nonregular.

(j) TRUE FALSE — If A is a non-context-free language, then A is also non-regular.

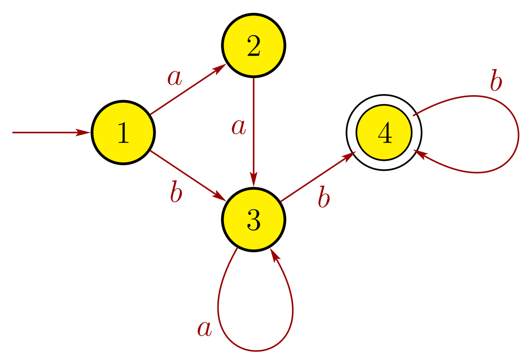
2. **[20 points]** Give short answers to each of the following parts. **Each answer should be at most a few sentences. Be sure to define any notation that you use.**

   (a) Let $\Sigma = \{0, 1\}$, and let A be the language of strings $w \in \Sigma^*$ containing exactly one double symbol. (A string has a *double symbol* if it contains 00 or 11 as a substring. The string 10010 has exactly one double symbol, but 100010 has two double symbols.) Give a regular expression for A.

   Answer: ____

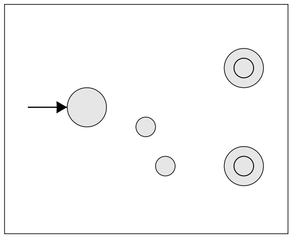
   (b) Give a regular expression for the language recognized by the NFA below.

   Answer: ____



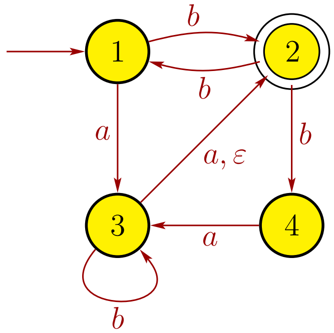
(c) Suppose that language A_1 is recognized by NFA N_1 below. Note that the transitions are not drawn in N_1. Draw a picture of an NFA for A_1^*.

N_1



(d) Suppose that A_1 is a language defined by a CFG $G_1 = (V_1, \Sigma, R_1, S_1)$, and A_2 is a language defined by a CFG $G_2 = (V_2, \Sigma, R_2, S_2)$, where the alphabet Σ is the same for both languages and $V_1 \cap V_2 = \emptyset$. Let $A_3 = A_1 \circ A_2$. Give a CFG G_3 for A_3 in terms of G_1 and G_2. You do not have to prove the correctness of your CFG G_3, but do not give just an example.

3. **[15 points]** Let N be the following NFA with $\Sigma = \{a, b\}$, and let $C = L(N)$.



Give a DFA for C.

Scratch-work area

4. **[30 points]** Consider the language

$$L = \{ c^i a^j b^k \mid i, j, k \geq 0, \text{ and } i = j \text{ or } j = k \}.$$

(a) Give a context-free grammar G for L. Be sure to specify G as a 4-tuple $G = (V, \Sigma, R, S)$.

(b) Give a PDA for L. You only need to draw the graph.

Scratch-work area

5. **[15 points]** Recall the pumping lemma for regular languages:

Theorem: If L is a regular language, then there exists a pumping length p where, if $s \in L$ with $|s| \geq p$, then s can be split into three pieces $s = xyz$ such that (i) $xy^iz \in L$ for each $i \geq 0$, (ii) $|y| \geq 1$, and (iii) $|xy| \leq p$.

Let $A = \{ c^i a^j b^k \mid i, j, k \geq 0, \text{ and } i = j \text{ or } j = k \}$. Is A a regular or nonregular language? If A is regular, give a regular expression for A. If A is not regular, prove that it is a nonregular language.

Circle one: **Regular Language** **Nonregular Language**