Midterm Exam 2
CS 341: Foundations of Computer Science II — **Fall 2015, day section**
Prof. Marvin K. Nakayama

Print family (or last) name: ____________________

Print given (or first) name: ____________________

I have read and understand all of the instructions below, and I will obey the University Code on Academic Integrity.

Signature and Date: ____________________

- This exam has 8 pages in total, numbered 1 to 8. Make sure your exam has all the pages.
- This exam will be 1 hour and 25 minutes in length.
- This is a closed-book, closed-note exam.
- For all problems, follow these instructions:
  1. Give only your answers in the spaces provided. I will only grade what you put in the answer space, and I will take off points for any scratch work in the answer space. Use the scratch-work area or the backs of the sheets to work out your answers before filling in the answer space.
  2. DFA stands for deterministic finite automaton; NFA stands for nondeterministic finite automaton; CFG stands for context-free grammar; PDA stands for pushdown automaton; TM stands for Turing machine.
  3. For any proofs, be sure to provide a step-by-step argument, with justifications for every step. Unless you are specifically asked to prove a theorem from the book, you may assume that the theorems in the textbook hold; i.e., you do not have to reprove the theorems in the textbook. When using a theorem from the textbook, make sure you provide enough detail so that it is clear which result you are using; e.g., say something like, "By the theorem that states $S^{**} = S^*$, it follows that ..."

Problem	1	2	3	4	5	6	Total
Points							

1. **[20 points]** For each of the following, circle TRUE if the statement is correct. Otherwise, circle FALSE

(a) TRUE FALSE — For any Turing machine $M = (Q, \Sigma, \Gamma, \delta, q_1, q_{\text{accept}}, q_{\text{reject}})$ and string $w \in \Sigma^*$, the TM M will either accept or reject w.

(b) TRUE FALSE — If a language is context-free, then it must be Turing-decidable.

(c) TRUE FALSE — The set of all Turing machines is countable.

(d) TRUE FALSE — The problem of determining if a context-free grammar generates the empty language is undecidable.

(e) TRUE FALSE — The universal Turing machine decides

$$A_{\text{TM}} = \{ \langle M, w \rangle \mid M \text{ is a TM that accepts string } w \}.$$

(f) TRUE FALSE — Every language is Turing-recognizable.

(g) TRUE FALSE — Two languages A and B are equal if $\overline{A} \cap B = \emptyset$.

(h) TRUE FALSE — The language

$$EQ_{\text{DFA}} = \{ \langle C, D \rangle \mid C \text{ and } D \text{ are DFAs with } L(C) = L(D) \}$$

is Turing-decidable.

(i) TRUE FALSE — The set of all infinite binary sequences is countable.

(j) TRUE FALSE — There are some languages recognized by a 5-tape, nondeterministic Turing machine that cannot be recognized by a 1-tape, deterministic Turing machine.

2. **[20 points]** Give a short answer (at most three sentences) for each part below. For parts (a), (b) and (c), let $A = \{x, y, z\}$ and $B = \{1, 2, 3\}$, and define the function $f : A \to B$ such that

$$\begin{aligned} f(x) &= 1, \\ f(y) &= 1, \\ f(z) &= 2. \end{aligned}$$

Explain your answers.

(a) Is f one-to-one?

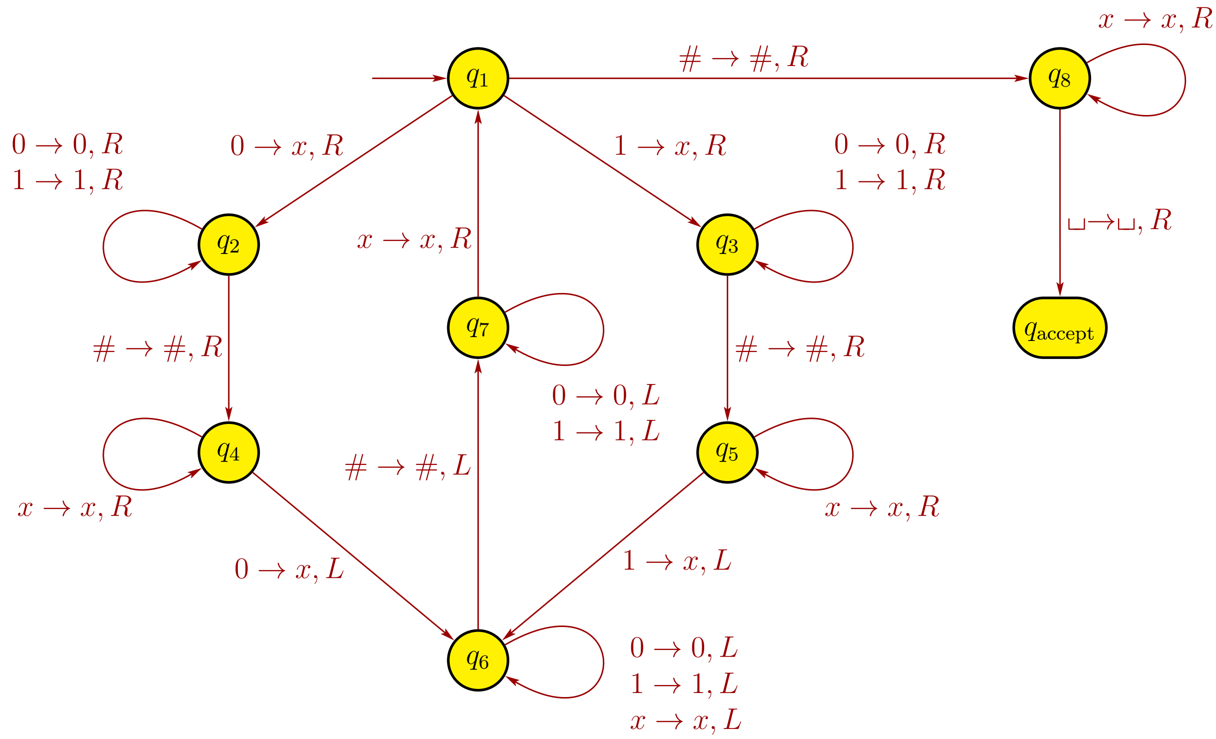
(b) Is f onto?

(c) Is f a correspondence?

(d) What is the difference between a Turing-recognizable language and a Turing-decidable language?

(e) What does the Church-Turing Thesis say?

3. **[10 points]** Consider the below Turing machine $M = (Q, \Sigma, \Gamma, \delta, q_1, q_{\text{accept}}, q_{\text{reject}})$ with $Q = \{q_1, \ldots, q_8, q_{\text{accept}}, q_{\text{reject}}\}$, $\Sigma = \{0, 1, \#\}$, $\Gamma = \{0, 1, \#, x, \sqcup\}$, and transitions below.



To simplify the figure, we don't show the reject state q_{reject} or the transitions going to the reject state. Those transitions occur implicitly whenever a state lacks an outgoing transition for a particular symbol. For example, because in state q_5 no outgoing arrow with a # is present, if a # occurs under the head when the machine is in state q_5, it goes to state q_{reject}. For completeness, we say that in each of these transitions to the reject state, the head writes the same symbol as is read and moves right.

Give the sequence of configurations that M enters when started on input string 010#1.

Each of the following problems requires you to prove a result. If you are asked to prove a result A and your proof relies on another result B, then you do not need to prove B if B is a result that we either went over in class or was in the homework. In this case, you need to make clear what result B you are citing in your proof of A (e.g., say something like, "By the result that $S^{**} = S^*$ for any set S of strings, we can show that ...").

4. **[15 points]** Let $\mathcal{L}$ be the set of all languages over an alphabet Σ. Prove $\mathcal{L}$ is uncountable.

5. [**20 points**] Consider the problem of determining **if a regular expression generates at least one string that ends in** 010. Express this problem as a language and show that it is decidable.

6. **[15 points]** Recall that

$$HALT_{\text{TM}} = \{ \langle M, w \rangle \mid M \text{ is a Turing machine that halts on input } w \}.$$

Prove that $HALT_{\text{TM}}$ is undecidable by showing that A_{TM} reduces to $HALT_{\text{TM}}$, where

$$A_{\text{TM}} = \{ \langle M, w \rangle \mid M \text{ is a Turing machine that accepts input } w \}.$$