CS 341, Spring 2015
Solutions for Midterm, eLearning Section

1. (a) True. Since A has a PDA, it is context-free by Theorem 2.20, so the statement then follows from Theorem 2.9.

   (b) True. The language $\emptyset$ is finite, so slide 1-95 shows that it is regular. Corollary 2.32 then implies that $\emptyset$ is also context-free.

   (c) False. For example, let A have regular expression $(0 \cup 1)^*$, so it is an infinite language. Since A has a regular expression, it is a regular language by Theorem 1.54.

   (d) True. By Corollary 1.40, A is regular since it has an NFA. Corollary 2.32 then implies that A is context-free, so it has a PDA by Theorem 2.20.

   (e) False. The language $A = \{ a^n b^n c^n \mid n \geq 0 \}$ is nonregular, which we can show by the pumping lemma for regular languages (to get a contradiction, choose the string $s = a^p b^p c^p \in A$, so $x = a^j$, $y = a^k$ and $z = a^l b^p c^p$, with $j + k + l = p$ and $k \geq 1$). But slide 2-96 shows that A is also non-context-free.

   (f) False. Let $A = \{ 0^n 1^n \mid n \geq 0 \}$ and B have regular expression $(0 \cup 1)^*$. Then B is regular since it has a regular expression (Theorem 1.54). Also, note that $A \subseteq B$, but A is nonregular, as shown on slide 1-105.

   (g) False. Homework 6, problem 2b.

   (h) True. Since A is finite, it is regular by slide 1-95. Thus, $\overline{A}$ is regular by Homework 2, problem 3. Also, B is regular since it has a regular expression (Theorem 1.54), so $\overline{A} \cap B$ is regular by Homework 2, problem 5. Hence, Corollary 2.32 implies $\overline{A} \cap B$ is context-free.

   (i) False. The derivation $S \Rightarrow 0$ generates the string 0, which is not in the language, so the CFG cannot be correct.

   (j) True. Homework 5, problem 3b.

2. (a) $a^* b (a \cup b a^* b)^*$

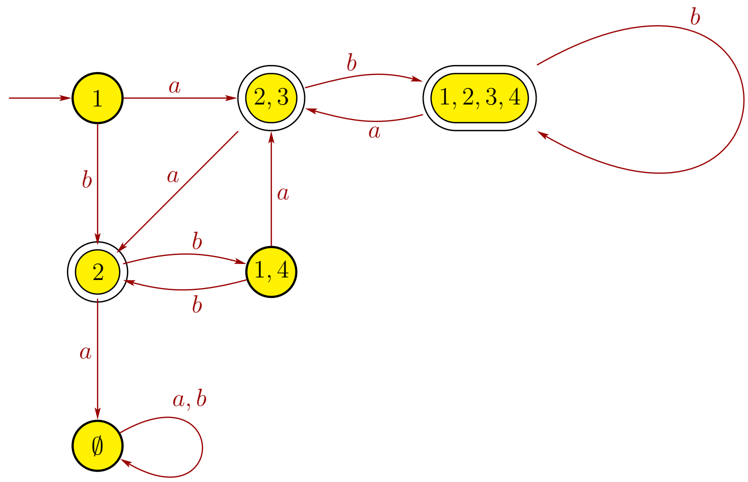
   (b)
   - $S \to XS$ is not in Chomsky normal form since starting variable cannot be on right side of rule.
   - $X \to Ya$ is improper since a rule cannot have a mix of terminals and variables on the right.
   - $Y \to \varepsilon$ is improper since ε cannot be on right side of rule unless S is on left side.
   - $Y \to YYXY$ is improper since a rule cannot have more than two variables on the right side.

   (c) slide 1-63.

   (d) Homework 5, problem 3c.

3. $q_1 1\#1$ $\quad x q_3 \#1$ $\quad x\# q_5 1$ $\quad x q_6 \# x$ $\quad q_7 x \# x$ $\quad x q_1 \# x$ $\quad x \# q_8 x$ $\quad x \# x q_8$ $\quad x \# x \sqcup q_{\text{accept}}$

4. Here's a DFA for C.

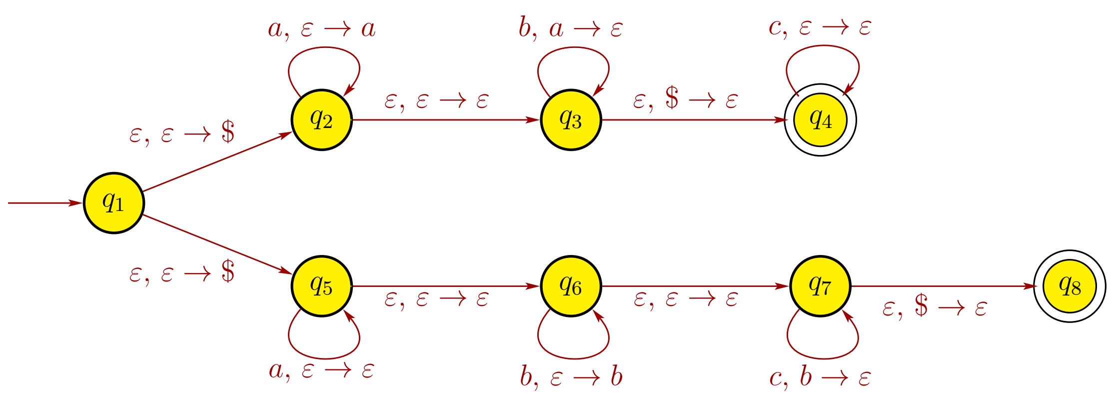


5. (a) $G = (V, \Sigma, R, S)$ with set of variables $V = \{S, U, W, X, Y\}$, where S is the start variable; set of terminals $\Sigma = \{a, b, c\}$; and rules

$$\begin{aligned} S &\rightarrow XY \mid UW \\ X &\rightarrow aXb \mid \varepsilon \\ Y &\rightarrow cY \mid \varepsilon \\ U &\rightarrow aU \mid \varepsilon \\ W &\rightarrow bWc \mid \varepsilon \end{aligned}$$

The rule $S \rightarrow XY$ eventually yields strings $a^i b^j c^k$ with $i = j$, and the rule $S \rightarrow UW$ eventually yields strings $a^i b^j c^k$ with $j = k$.

(b) PDA



The PDA has a nondeterministic branch at q_1. If the string is $a^i b^j c^k$ with $i = j$, then the PDA takes the branch from q_1 to q_2. If the string is $a^i b^j c^k$ with $j = k$, then the PDA takes the branch from q_1 to q_5.

There are other correct PDAs that recognize A.

6. This is Homework 2, problem 4. We prove this by contradiction. Suppose that $\overline{M}$ is not a minimal DFA for $\overline{A}$. Then there exists another DFA D for $\overline{A}$ such that D has

strictly fewer states than $\overline{M}$. Now create another DFA D' by swapping the accepting and non-accepting states of D. Then D' recognizes the complement of $\overline{A}$. But the complement of $\overline{A}$ is just A, so D' recognizes A. Note that D' has the same number of states as D, and $\overline{M}$ has the same number of states as M. Thus, since we assumed that D has strictly fewer states than $\overline{M}$, then D' has strictly fewer states than M. But since D' recognizes A, this contradicts our assumption that M is a minimal DFA for A. Therefore, $\overline{M}$ is a minimal DFA for $\overline{A}$.

7. Suppose that A is a regular language. Let p be the pumping length, and consider the string $s = a^p b^p$. Note that $s \in A$ since the numbers of a's and b's are equal. Also, $|s| = 2p \geq p$, so the pumping lemma implies we can write $s = xyz$ with $xy^i z \in A$ for all $i \geq 0$, $|y| > 0$, and $|xy| \leq p$. Now, $|xy| \leq p$ implies that x and y have only a's (together up to p in total) and z has the rest of the a's at the beginning, followed by b^p. Hence, we can write $x = a^j$ for some $j \geq 0$, $y = a^k$ for some $k \geq 0$, and $z = a^\ell b^p$, where $j + k + \ell = p$ since $xyz = s = a^p b^p$. Also, $|y| > 0$ implies $k > 0$. Now consider the string $xyyz = a^j a^k a^k a^\ell b^p = a^{p+k} b^p$ since $j + k + \ell = p$. Since $k > 0$, the number of a's and b's are not equal. Also, the number of c's, which is 0, does not equal the number of b's, so $xyyz \notin A$. This contradicts (i), so A is not a regular language.