Midterm Exam
CS 341-452: Foundations of Computer Science II — **Spring 2015, eLearning section**
Prof. Marvin K. Nakayama

Print family (i.e., last) name: _____

Print given (i.e., first) name: _____

I have read and understand all of the instructions below, and I will obey the University Code on Academic Integrity.

_____
Signature and Date

- This exam has 9 pages in total, numbered 1 to 9. Make sure your exam has all the pages.

- Unless other prior arrangements have been made with the professor, the exam is to last 2.5 hours and is to be given on Saturday, March 7, 2015, 9:30am–12:00pm.

- This is a closed-book, closed-note exam. No electronic devices (e.g., calculators, cellphones) are allowed.

- For all problems, follow these instructions:

  1. Give only your answers in the spaces provided. I will only grade what you put in the answer space, and I will take off points for any scratch work in the answer space. Use the scratch-work area or the backs of the sheets to work out your answers before filling in the answer space.

  2. DFA stands for deterministic finite automaton; NFA stands for nondeterministic finite automaton; CFG stands for context-free grammar; PDA stands for pushdown automaton. TM stands for Turing machine.

  3. For any proofs, be sure to provide a step-by-step argument, with justifications for every step. If you are asked to prove a result X, then in your proof of X, you may use any other result Y without proving Y. However, make it clear what the other result Y is that you are using; e.g., write something like, "By the result that $A^{**} = A^*$, we know that ...."

Problem	1	2	3	4	5	6	7	Total
Points								

1. [**20 points**] For each of the following, circle TRUE if the statement is correct. Otherwise, circle FALSE

    (a)   TRUE   FALSE — If A has a PDA, then A must have a CFG in Chomsky normal form.

    (b)   TRUE   FALSE — $\emptyset$ is a context-free language.

    (c)   TRUE   FALSE — If A is a regular language, then A is finite.

    (d)   TRUE   FALSE — If A has an NFA, then A must have a PDA.

    (e)   TRUE   FALSE — Every nonregular language is context-free.

    (f)   TRUE   FALSE — If A and B are languages such that $A \subseteq B$ and B is regular, then A must be regular.

    (g)   TRUE   FALSE — The class of context-free languages is closed under complements.

    (h)   TRUE   FALSE — If A is a finite language and B has a regular expression, then $\overline{A} \cap B$ must be context-free.

    (i)   TRUE   FALSE — The language $\{\, 1^n 0^n \mid n \geq 0 \,\}$ has context-free grammar $G = (V, \Sigma, R, S)$, with $V = \{S\}$, $\Sigma = \{0, 1\}$, start variable S, and rules $S \rightarrow 1S0 \mid 0$.

    (j)   TRUE   FALSE — The class of context-free languages is closed under concatenation.

2. **[20 points]** Give short answers to each of the following parts. **Each answer should be at most a few sentences. Be sure to define any notation that you use.**

(a) Let $\Sigma = \{a, b\}$, and let A be the set of strings over Σ that have an odd number of b's. Give a regular expression for A.

(b) Consider the following CFG $G = (V, \Sigma, R, S)$, with $V = \{S, X, Y\}$, $\Sigma = \{a, b\}$, start variable S, and rules R as follows:

$$
\begin{aligned}
S &\rightarrow XS \mid YY \mid a \mid \varepsilon \\
X &\rightarrow Ya \mid b \\
Y &\rightarrow YX \mid \varepsilon \mid YYXY
\end{aligned}
$$

Note that G is not in Chomsky normal form. List all of the rules in G that violate Chomsky normal form. Explain your answer.

(c) Suppose that language A_1 is recognized by NFA N_1 below, and language A_2 is recognized by NFA N_2 below. Note that the transitions are not drawn in N_1 and N_2. Draw a picture of an NFA for $A_1 \circ A_2$.

N_1

N_2
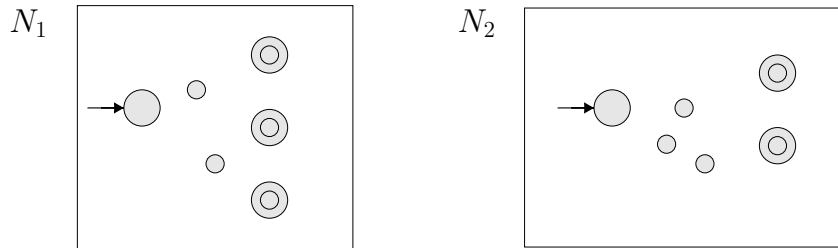
(d) Suppose that A is a language defined by a CFG $G_1 = (V_1, \Sigma, R_1, S_1)$. Give a CFG G_2 for A^* in terms of G_1. You do not have to prove the correctness of your CFG G_2, but do not give just an example.

3. **[10 points]**　Consider the below Turing machine $M = (Q, \Sigma, \Gamma, \delta, q_1, q_{\text{accept}}, q_{\text{reject}})$ with $Q = \{q_1, \ldots, q_8, q_{\text{accept}}, q_{\text{reject}}\}$, $\Sigma = \{0, 1, \#\}$, $\Gamma = \{0, 1, \#, x, \sqcup\}$, and transitions below.
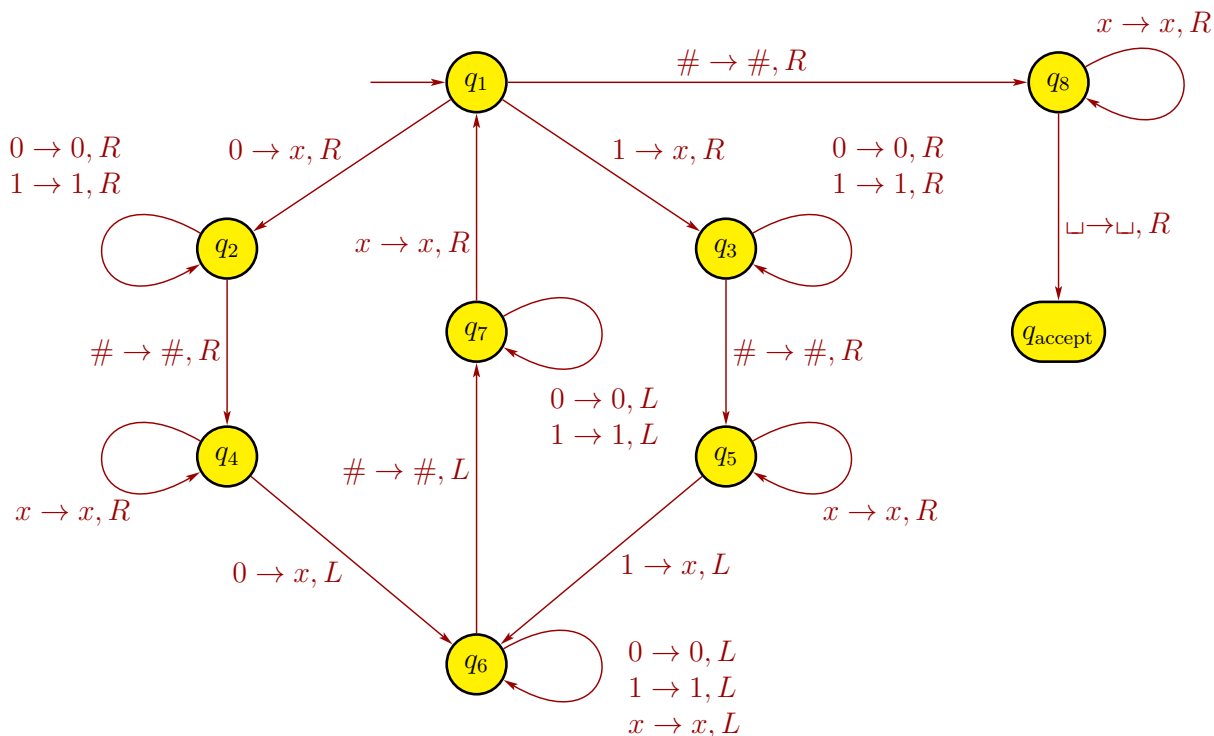
To simplify the figure, we don't show the reject state q_{reject} or the transitions going to the reject state. Those transitions occur implicitly whenever a state lacks an outgoing transition for a particular symbol. For example, because in state q_5 no outgoing arrow with a $\#$ is present, if a $\#$ occurs under the head when the machine is in state q_5, it goes to state q_{reject}. For completeness, we say that in each of these transitions to the reject state, the head writes the same symbol as is read and moves right.

Give the sequence of configurations that M enters when started on the input string $1\#1$.

4. **[10 points]** Let N be the following NFA with $\Sigma = \{a, b\}$, and let $C = L(N)$.
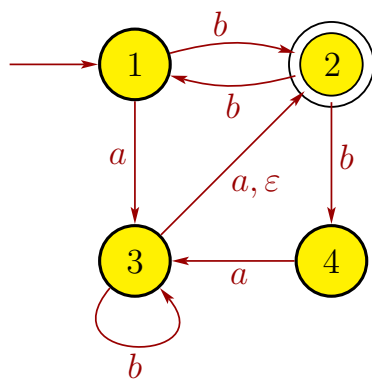
Give a DFA for C.

5. **[20 points]** Let $\Sigma = \{a, b, c\}$, and consider the language $A = \{a^i b^j c^k \mid i, j, k \geq 0, \text{ and } i = j \text{ or } j = k \}$.

(a) Give a CFG G for A. Be sure to specify G as a 4-tuple $G = (V, \Sigma, R, S)$.

(b) Give a PDA for A. You only need to give the drawing.

Scratch-work area

Each of the following problems may require you to prove a result. Unless stated otherwise, in your proofs, you can apply any theorems or results that we went over in class (lectures, homeworks) without proving them, except for the result you are asked to prove in the problem. When citing a theorem or result, make sure that you give enough details so that it is clear what theorem or result you are using (e.g., say something like, "By the result that says $A^{**} = A^*$, we can show that ....")

6. [**10 points**]   We say that a DFA M for a language A is *minimal* if there does not exist another DFA M' for A such that M' has strictly fewer states than M. Suppose that $M = (Q, \Sigma, \delta, q_0, F)$ is a minimal DFA for A. Using M, we construct a DFA $\overline{M}$ for the complement $\overline{A}$ as $\overline{M} = (Q, \Sigma, \delta, q_0, Q - F)$. Prove that $\overline{M}$ is a minimal DFA for $\overline{A}$.

7. [**10 points**]   Recall the pumping lemma for regular languages:

Theorem: If L is a regular language, then there is a number p (pumping length) where, if $s \in L$ with $|s| \geq p$, then s can be split into 3 pieces, $s = xyz$, satisfying conditions

(i) $xy^i z \in L$ for each $i \geq 0$,

(ii) $|y| > 0$, and

(iii) $|xy| \leq p$.

Let $\Sigma = \{a, b, c\}$, and consider the language $A = \{a^i b^j c^k \mid i, j, k \geq 0, \text{ and } i = j \text{ or } j = k\}$. Prove that A is not a regular language.