CS 341, Fall 2016
Solutions for Midterm, eLearning Section

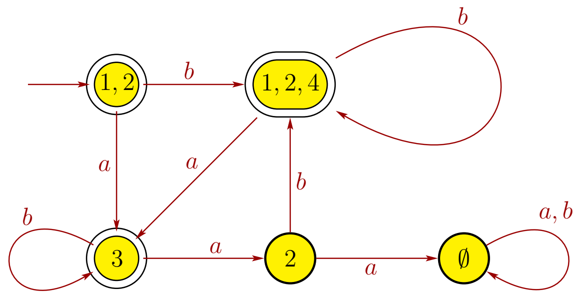
1. (a) True. Suppose A is non-context-free but regular. But then Corollary 2.32 implies A is context-free, which is a contradiction.
   (b) False. The language a^* is regular but infinite.
   (c) False. The TM M can also loop on w.
   (d) False. $\{a^n b^n c^n \mid n \geq 0\}$ is non-regular, but not context-free.
   (e) False. HW 6, problem 2(a).
   (f) True. Kleene's Theorem ensures A^* is regular, and we know $\overline{B}$ is regular by HW 2, problem 3. Thus, $A^* \cap \overline{B}$ is regular by HW 2, problem 5.
   (g) True. Corollary 2.32 implies A is context-free. Thus, A has a PDA by Theorem 2.20.
   (h) False. For example, $A = \emptyset$ is a subset of $B = \{0^n 1^n \mid n \geq 0\}$, but A is regular and B is non-regular.
   (i) False. Theorem 1.39.
   (j) True. By Theorem 2.9. The fact that A is non-regular is irrelevant.

2. (a) $b^* a (b \cup ab^* a)^*$
   (b)
   - $X \to Ya$ is improper since a rule cannot have a mix of terminals and variables on the right.
   - $Y \to XS$ is not in Chomsky normal form since starting variable cannot be on right side of rule.
   - $Y \to \varepsilon$ is improper since ε cannot be on right side of rule unless S is on left side.
   - $Y \to YYXY$ is improper since a rule cannot have more than two variables on the right side.

   (c) A DFA $M_3 = (Q_3, \Sigma, \delta_3, q_3, F_3)$ for $A_1 \cap A_2$ has the set of states as $Q_3 = Q_1 \times Q_2$, alphabet Σ, start state $q_3 = (q_1, q_2) \in Q_3$, the set of accepting states as $F_3 = F_1 \times F_2$, and transition function $\delta_3((x, y), \ell) = (\delta_1(x, \ell), \delta_2(y, \ell))$ for $x \in Q_1$, $y \in Q_2$, and $\ell \in \Sigma$.
   (d) The is Homework 5, problem 3c. Given a CFG $G_1 = (V_1, \Sigma, R_1, S_1)$ for a language A, a CFG $G_2 = (V_2, \Sigma, R_2, S_2)$ for A^* has $V_2 = V_1 \cup \{S_2\}$, where $S_2 \notin V_1$ is the new start variable, the same alphabet Σ as G_1, and rules $R_2 = R_1 \cup \{S_2 \to S_1 S_2, S_2 \to \varepsilon\}$.

3. $q_1 01\#0$ $\quad xq_2 1\#0$ $\quad x1q_2\#0$ $\quad x1\#q_4 0$ $\quad x1q_6\#x$ $\quad xq_7 1\#x$ $\quad q_7 x1\#x$ $\quad xq_1 1\#x$ $\quad xxq_3\#x$ $\quad xx\#q_5 x$ $\quad xx\#xq_5 \sqcup$ $\quad xx\#x \sqcup q_{\text{reject}} \sqcup$

4. Here's a DFA for C.

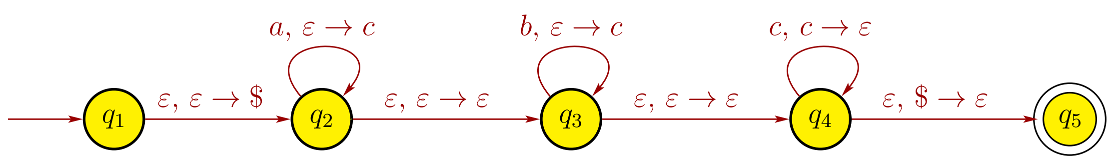


5. (a) $G = (V, \Sigma, R, S)$ with set of variables $V = \{S, X\}$, where S is the start variable; set of terminals $\Sigma = \{a, b, c\}$; and rules

$$\begin{aligned} S &\to aSc \mid X \\ X &\to bXc \mid \varepsilon \end{aligned}$$

The rule $S \to aSc$ eventually yields $a^i X c^i$ for $i \geq 0$. Then applying $X \to bXc$ j times and then $X \to \varepsilon$ leads to $a^i b^j c^j c^i = a^i b^j c^{i+j}$.

(b) PDA

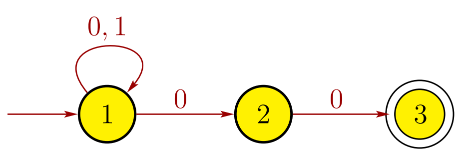


The PDA first reads each a, and pushes a c on the stack for each a. Then the PDA reads each b, and pushes a c on the stack for each b. Then the PDA reads each c, popping the stack each time. The last transition from q_4 to q_5 makes sure the number of c's equals the sum of the a's and b's.

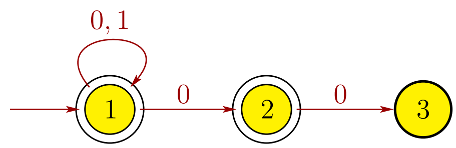
There are other correct PDAs that recognize A.

6. This is Homework 3, problem 2.

   (a) The NFA M below recognizes the language $C = \{\, w \in \Sigma^* \mid w \text{ ends with } 00 \,\}$, where $\Sigma = \{0, 1\}$.



Swapping the accept and non-accept states of M gives the following NFA M':



Note that M' accepts the string $100 \notin \overline{C} = \{ w \mid w \text{ does not end with } 00 \}$, so M' does not recognize the language $\overline{C}$.

(b) The class of languages recognized by NFAs is closed under complement, which we can prove as follows. Suppose that C is a language recognized by some NFA M, i.e., $C = L(M)$. Since every NFA has an equivalent DFA (Theorem 1.19), there is a DFA D such that $L(D) = L(M) = C$. By problem 3 on Homework 2, we then know there is another DFA $\overline{D}$ that recognizes the language $\overline{L(D)}$. Since every DFA is also an NFA, this then shows that there is an NFA, in particular $\overline{D}$, that recognizes the language $\overline{C} = \overline{L(D)}$. Thus, the class of languages recognized by NFAs is closed under complement.

7. The language A is nonregular. To prove this, suppose that A is a regular language. Let p be the pumping length, and consider the string $s = a^p c^p$. Note that $s \in A$ since the number of c's equals the sum of the a's and b's. Also, $|s| = 2p \geq p$, so the pumping lemma implies we can write $s = xyz$ with $xy^i z \in A$ for all $i \geq 0$, $|y| > 0$, and $|xy| \leq p$. Now, $|xy| \leq p$ implies that x and y have only a's (together up to p in total) and z has the rest of the a's at the beginning, followed by c^p. Hence, we can write $x = a^j$ for some $j \geq 0$, $y = a^k$ for some $k \geq 0$, and $z = a^\ell c^p$, where $j + k + \ell = p$ since $xyz = s = a^p c^p$. Also, $|y| > 0$ implies $k > 0$. Now consider the string $xyyz = a^j a^k a^k a^\ell c^p = a^{p+k} c^p$ since $j + k + \ell = p$. Since $k > 0$, the number of a's plus b's is $p + k$, which does not equal the number of c's, which is p. This contradicts (i), so A is not a regular language.