Midterm Exam
CS 341-451: Foundations of Computer Science II — **Fall 2016, eLearning section**
Prof. Marvin K. Nakayama

Print family (or last) name: _____

Print given (or first) name: _____

I have read and understand all of the instructions below, and I will obey the University Code on Academic Integrity.

_____
Signature and Date

- This exam has 9 pages in total, numbered 1 to 9. Make sure your exam has all the pages.
- Note the number written on the upper right-hand corner of the first page. On the sign-up sheet being passed around, print your name next to this number.
- Unless other prior arrangements have been made with the professor, the exam is to last 2.5 hours and is to be given on Saturday, October 22, 2016.
- This is a closed-book, closed-note exam. Electronic devices (e.g., calculators, cellphones, smart watches) are not allowed.
- For all problems, follow these instructions:
  1. Give only your answers in the spaces provided. I will only grade what you put in the answer space, and I will take off points for any scratch work in the answer space. Use the scratch-work area or the backs of the exam sheets to work out your answers before filling in the answer space.
  2. DFA stands for deterministic finite automaton; NFA stands for nondeterministic finite automaton; CFG stands for context-free grammar; PDA stands for pushdown automaton. TM stands for Turing machine.
  3. For any proofs, be sure to provide a step-by-step argument, with justifications for every step. If you are asked to prove a result X, you may use in your proof of X any other result Y without proving Y. However, make it clear what the other result Y is that you are using; e.g., write something like, "By the result that $A^{**} = A^*$, we know that ...."

Problem	1	2	3	4	5	6	7	Total
Points								

1. **[20 points]** For each of the following, circle TRUE if the statement is correct. Otherwise, circle FALSE

(a) TRUE FALSE — Every non-context-free language is also non-regular.

(b) TRUE FALSE — If A is a regular language, then A is finite.

(c) TRUE FALSE — If $M = (Q, \Sigma, \Gamma, \delta, q_0, q_{\text{accept}}, q_{\text{reject}})$ is a Turing machine and $w \in \Sigma^*$ is a string, then M either accepts or rejects w.

(d) TRUE FALSE — Every nonregular language is context-free.

(e) TRUE FALSE — The class of context-free languages is closed under intersection.

(f) TRUE FALSE — If A and B are regular languages, then so is $A^* \cap \overline{B}$.

(g) TRUE FALSE — If language A has a regular expression, then A has a PDA.

(h) TRUE FALSE — If $A \subseteq B$ and A is a regular language, then B is a regular language.

(i) TRUE FALSE — There is a language recognized by an NFA but has no DFA.

(j) TRUE FALSE — If A is a context-free language that is also non-regular, then A has a CFG in Chomsky normal form.

2. **[20 points]** Give short answers to each of the following parts. **Each answer should be at most a few sentences. Be sure to define any notation that you use.**

   (a) Let $\Sigma = \{a, b\}$, and let A be the set of strings over Σ that have an odd number of a's. Give a regular expression for A.

   (b) Consider the following CFG $G = (V, \Sigma, R, S)$, with $V = \{S, X, Y\}$, $\Sigma = \{a, b\}$, start variable S, and rules R as follows:

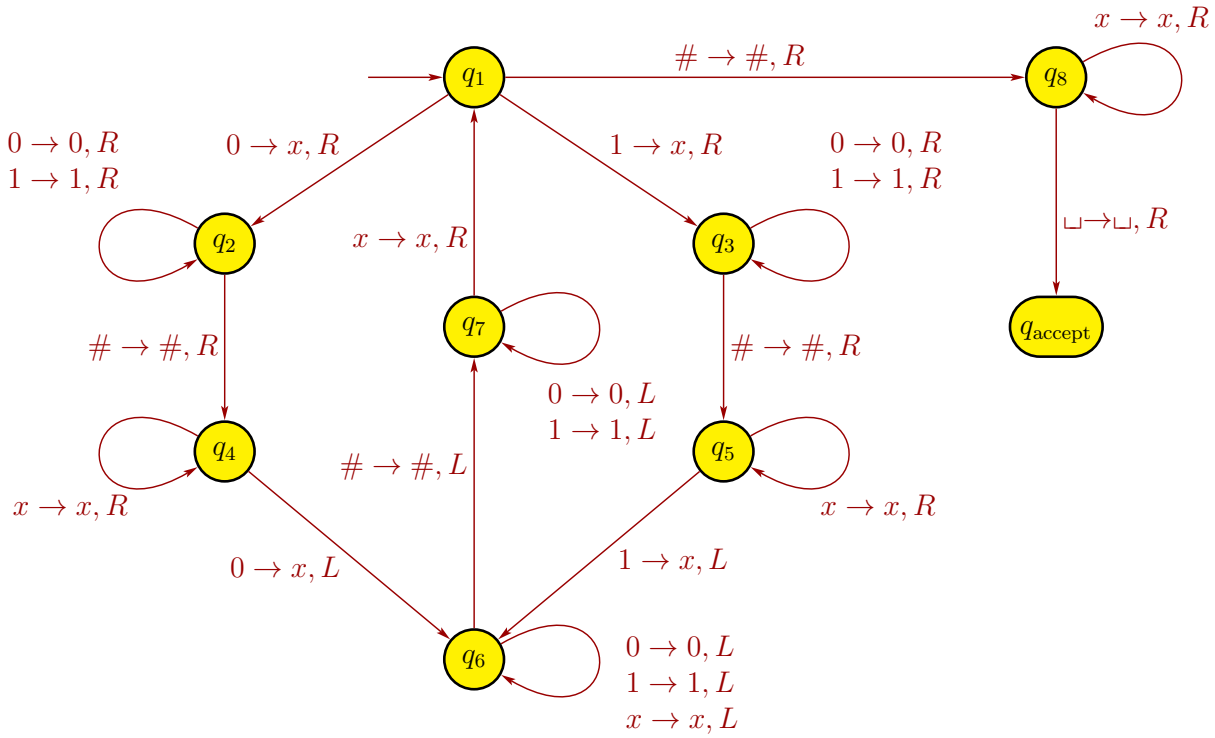
$$\begin{aligned} S &\rightarrow YY \mid a \mid \varepsilon \\ X &\rightarrow Ya \mid b \\ Y &\rightarrow YX \mid XS \mid \varepsilon \mid YYXY \end{aligned}$$

   Note that G is not in Chomsky normal form. List all of the rules in G that violate Chomsky normal form. Explain your answer.

(c) Let $M_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$ be a DFA with language A_1, and $M_2 = (Q_2, \Sigma, \delta_2, q_2, F_2)$ be a DFA with language A_2. Consider the language $A = A_1 \cap A_2$. Give a DFA M_3 for A in terms of M_1 and M_2. Your DFA M_3 must be completely general. Do not prove the correctness of your DFA M_3, but do not just give an example.

(d) Suppose that A is a language defined by a CFG $G_1 = (V_1, \Sigma, R_1, S_1)$. Give a CFG G_2 for A^* in terms of G_1. You do not have to prove the correctness of your CFG G_2, but do not give just an example.

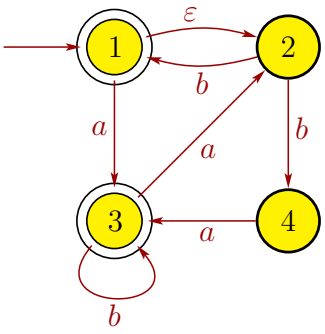
3. **[10 points]** Consider the below Turing machine $M = (Q, \Sigma, \Gamma, \delta, q_1, q_{\text{accept}}, q_{\text{reject}})$ with $Q = \{q_1, \ldots, q_8, q_{\text{accept}}, q_{\text{reject}}\}$, $\Sigma = \{0, 1, \#\}$, $\Gamma = \{0, 1, \#, x, \sqcup\}$, and transitions below.



To simplify the figure, we don't show the reject state q_{reject} or the transitions going to the reject state. Those transitions occur implicitly whenever a state lacks an outgoing transition for a particular symbol. For example, because in state q_5 no outgoing arrow with a # is present, if a # occurs under the head when the machine is in state q_5, it goes to state q_{reject}. For completeness, we say that in each of these transitions to the reject state, the head writes the same symbol as is read and moves right.

Give the sequence of configurations that M enters when started on the input string 01#0.

4. **[10 points]** Let N be the following NFA with $\Sigma = \{a, b\}$, and let $C = L(N)$.



Give a DFA for C.

5. **[20 points]** Let $\Sigma = \{a, b, c\}$, and consider the language $A = \{a^i b^j c^k \mid i, j, k \geq 0, \text{ and } i + j = k\}$.

   (a) Give a CFG G for A. Be sure to specify G as a 4-tuple $G = (V, \Sigma, R, S)$.

   (b) Give a PDA for A. You only need to give the drawing.

Scratch-work area

Each of the following problems may require you to prove a result. Unless stated otherwise, in your proofs, you can apply any theorems or results that we went over in class (lectures, homeworks) without proving them, except for the result you are asked to prove in the problem. When citing a theorem or result, make sure that you give enough details so that it is clear what theorem or result you are using (e.g., say something like, "By the result that says $A^{**} = A^*$, we can show that ....")

6. [**10 points**]

   (a) Show by giving an example that, if M is an NFA that recognizes language C, swapping the accept and non-accept states in M doesn't necessarily yield a new NFA that recognizes $\overline{C}$. Explain your answer.

   (b) Is the class of languages recognized by NFAs closed under complementation? Explain your answer.

7. [**10 points**] Recall the pumping lemma for regular languages:

Theorem: If L is a regular language, then there is a number p (pumping length) where, if $s \in L$ with $|s| \geq p$, then s can be split into 3 pieces, $s = xyz$, satisfying conditions

(i) $xy^iz \in L$ for each $i \geq 0$,

(ii) $|y| > 0$, and

(iii) $|xy| \leq p$.

Let $\Sigma = \{a, b, c\}$, and consider the language $A = \{\, a^i b^j c^k \mid i, j, k \geq 0, \text{ and } i + j = k \,\}$. Is A a regular or nonregular language? If A is regular, give a regular expression for A. If A is not regular, prove that it is a nonregular language.

Circle one: **Regular Language** **Nonregular Language**