Midterm Exam 2
CS 341: Foundations of Computer Science II — **Fall 2016, face-to-face day section**
Prof. Marvin K. Nakayama

Print family (or last) name: ______________________________

Print given (or first) name: ______________________________

I have read and understand all of the instructions below, and I will obey the University Code on Academic Integrity.

Signature and Date: ______________________________

- This exam has 9 pages in total, numbered 1 to 9. Make sure your exam has all the pages.
- This exam will be 1 hour and 25 minutes in length.
- This is a closed-book, closed-note exam. Electronic devices (e.g., cellphone, smart watch, calculator) are not allowed.
- For all problems, follow these instructions:
  1. Give only your answers in the spaces provided. I will only grade what you put in the answer space, and I will take off points for any scratch work in the answer space. Use the scratch-work area or the backs of the sheets to work out your answers before filling in the answer space.
  2. DFA stands for deterministic finite automaton; NFA stands for nondeterministic finite automaton; CFG stands for context-free grammar; PDA stands for pushdown automaton; TM stands for Turing machine.
  3. For any proofs, be sure to provide a step-by-step argument, with justifications for every step. Unless you are specifically asked to prove a theorem from the book or notes, you may assume that the theorems in the textbook and notes hold; i.e., you do not have to reprove the theorems in the textbook and notes. When using a theorem from the textbook or notes, make sure you provide enough detail so that it is clear which result you are using; e.g., say something like, "By the theorem that states $S^{**} = S^*$, it follows that . . . "

Problem	1	2	3	4	5	6	7	Total
Points								

1. **[20 points]** For each of the following, circle TRUE if the statement is correct. Otherwise, circle FALSE

(a) TRUE FALSE — For any Turing machine $M = (Q, \Sigma, \Gamma, \delta, q_1, q_{\text{accept}}, q_{\text{reject}})$ and string $w \in \Sigma^*$, M will either accept or reject w.

(b) TRUE FALSE — Every language is Turing-recognizable.

(c) TRUE FALSE — Every Turing-decidable language is also Turing-recognizable.

(d) TRUE FALSE — Every multi-tape Turing machine has an equivalent single-tape Turing machine.

(e) TRUE FALSE — The universal Turing machine recognizes

$$A_{\text{TM}} = \{\, \langle M, w \rangle \mid M \text{ is a TM that accepts string } w \,\}.$$

(f) TRUE FALSE — Two languages A and B are equal if and only if $A \cap \overline{B} = \emptyset$.

(g) TRUE FALSE — Every infinite set is uncountable.

(h) TRUE FALSE — Every regular language is Turing-decidable.

(i) TRUE FALSE — The set of all Turing machines is countable.

(j) TRUE FALSE — There is a language A that is recognized by a nondeterministic Turing machine but is not recognized by any deterministic Turing machine.

2. **[20 points]** Give a short answer (at most three sentences) for each part below. For parts (a), (b) and (c), let $A = \{x, y, z\}$ and $B = \{1, 2\}$, and define the function $f : A \to B$ such that

$$\begin{aligned} f(x) &= 1, \\ f(y) &= 2, \\ f(z) &= 1. \end{aligned}$$

Explain your answers.

(a) Is f one-to-one?

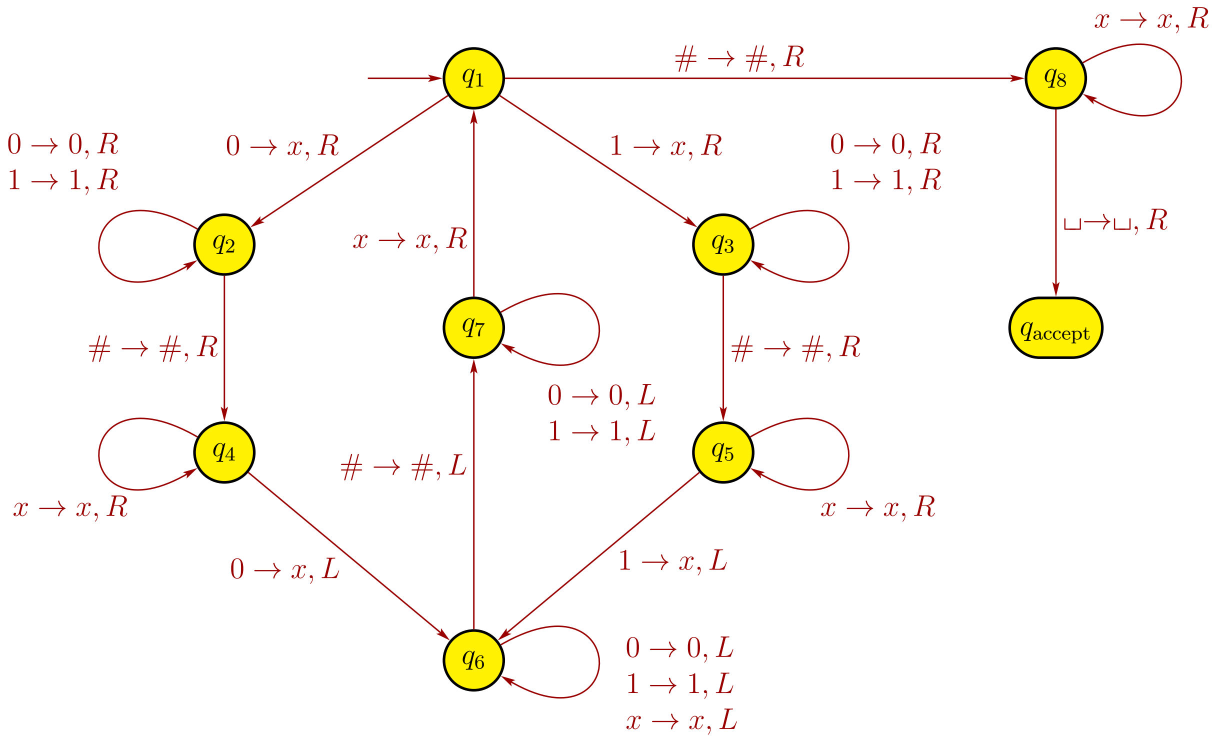
(b) Is f onto?

(c) Is f a correspondence?

(d) What is the difference between a Turing-recognizable language and a Turing-decidable language?

(e) What does the Church-Turing Thesis say?

3. **[10 points]** Consider the below Turing machine $M = (Q, \Sigma, \Gamma, \delta, q_1, q_{\text{accept}}, q_{\text{reject}})$ with $Q = \{q_1, \ldots, q_8, q_{\text{accept}}, q_{\text{reject}}\}$, $\Sigma = \{0, 1, \#\}$, $\Gamma = \{0, 1, \#, x, \sqcup\}$, and transitions below.



To simplify the figure, we don't show the reject state q_{reject} or the transitions going to the reject state. Those transitions occur implicitly whenever a state lacks an outgoing transition for a particular symbol. For example, because in state q_5 no outgoing arrow with a # is present, if a # occurs under the head when the machine is in state q_5, it goes to state q_{reject}. For completeness, we say that in each of these transitions to the reject state, the head writes the same symbol as is read and moves right.

Give the sequence of configurations that M enters when started on the input string 0#0.

Each of the following problems requires you to prove a result. Unless stated otherwise, in your proofs, you can apply any theorems that we went over in class without proving them, except for the result you are asked to prove in the problem. When citing a theorem, make sure that you give enough details so that it is clear what theorem you are using (e.g., say something like, "By the theorem that says every context-free language has a CFG in Chomsky normal form, we can show that ....")

4. **[10 points]** Let $\mathcal{B}$ be the set of all infinite sequences over $\{0, 1\}$. Show that $\mathcal{B}$ is uncountable.

5. **[15 points]** Consider the problem of determining **if a regular expression generates at least one string that has** 010 **as a substring**. Express this problem as a language and show that it is decidable.

6. **[15 points]** Prove that a language A is decidable if and only if it is both Turing-recognizable and co-Turing-recognizable.

7. **[10 points]** Recall that

$$HALT_{\text{TM}} = \{ \langle M, w \rangle \mid M \text{ is a Turing machine that halts on input } w \}.$$

Prove that $HALT_{\text{TM}}$ is undecidable.