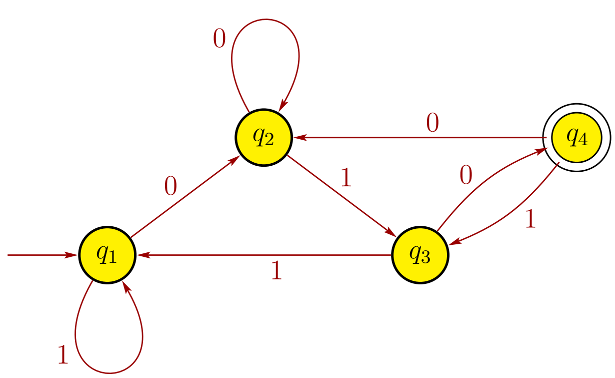
CS 341, Fall 2018
Solutions for Midterm, eLearning Section

1. (a) True. Because A is finite, it is regular by the theorem on slide 1-95 of the notes. Corollary 2.32 then ensures that A is regular.
   (b) False. For example, let $A = \{ a^n b^n \mid n \geq 0 \}$ and B have regular expression $(a \cup b)^*$. Then $A \subseteq B$, B is regular, but A is nonregular.
   (c) False. By Homework 6, problem 2(b).
   (d) False. $A = \{ a^n b^n c^n \mid n \geq 0 \}$ is nonregular and not context-free.
   (e) False. The language with regular expression a^* is regular but infinite.
   (f) False. a^*b^* generates the string $abb \notin \{a^n b^n \mid n \geq 0\}$. In fact, the language $\{a^n b^n \mid n \geq 0\}$ is not regular, so it does not have a regular expression.
   (g) True. By Theorem 2.9. The fact that A is non-regular is irrelevant.
   (h) False. For example, let $A = \{abc\}$ and $B = \{a^n b^n c^n \mid n \geq 0\}$, so $A \subseteq B$. Because A is finite, it is regular (slide 1-95), so it is also context-free by Corollary 2.32. But B is not context-free by slide 2-96.
   (i) True. Because A has an NFA, it is regular by Corollary 1.40. As B is finite, it is regular by slide 1-95, and we know $\overline{B}$ is regular by HW 2, problem 3. Thus, $A \cap \overline{B}$ is regular by HW 2, problem 5, so Corollary 2.32 implies $A \cap \overline{B}$ is context-free.
   (j) False. A TM can loop on w.

2. (a) This is essentially the same as HW 3, problem 4c. A regular expression is $b^*ab^*ab^* \cup a^*ba^*b(a \cup b)^*$.
   (b) After the one step to remove $A \to \varepsilon$, the CFG is then

$$\begin{aligned} S_0 &\to S \\ S &\to bXSa \mid bSa \mid SaXbXS \mid SabXS \mid SaXbS \mid SabS \mid \varepsilon \\ A &\to aSb \end{aligned}$$

   (c) This is essentially the same as HW 2, problem 2c.

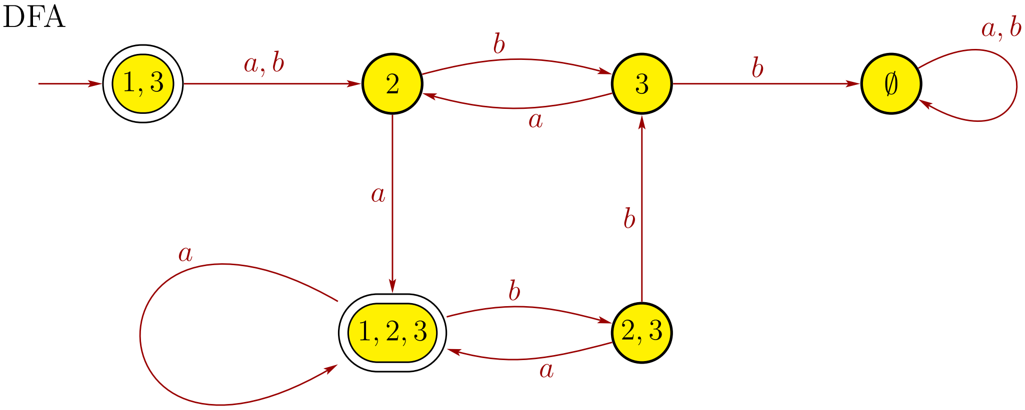


(d) Homework 5, problem 3a.

3. $q_1bb \quad \sqcup q_2b \quad \sqcup xq_3\sqcup \quad \sqcup q_5x \quad q_5\sqcup x \quad \sqcup q_2x \quad \sqcup xq_2\sqcup \quad \sqcup x\sqcup q_{\text{accept}}$
It is also correct to write the third configuration $\sqcup xq_3\sqcup$ as $\sqcup xq_3$ instead.

4. Homework 3, problem 2.

5. DFA

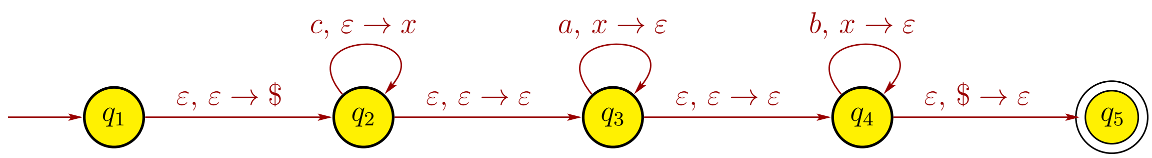


6. (a) $G = (V, \Sigma, R, S)$, with $V = \{S, X\}$, $\Sigma = \{a, b, c\}$, start variable S and rules

$$S \rightarrow cSb \mid X$$
$$X \rightarrow cXa \mid \varepsilon$$

There are infinitely many other correct CFGs for A.

(b) PDA



For every c read in the first part of the string, the PDA pushes an x onto the stack. Then the a's and the b's read pop the x's off the stack.

There are infinitely many other correct PDAs for A.

7. The language A is not regular. To prove this, suppose that A is a regular language. Let p be the pumping length, and consider the string $s = a^pba^p \in A$. Note that $|s| = 2p + 1 \geq p$, so the pumping lemma implies we can write $s = xyz$ with $xy^iz \in A$ for all $i \geq 0$, $|y| > 0$, and $|xy| \leq p$. Now, $|xy| \leq p$ implies that x and y have only a's (together up to p in total) and z has the rest of the a's at the beginning, followed by ba^p. Hence, we can write $x = a^j$ for some $j \geq 0$, $y = a^k$ for some $k \geq 0$, and $z = a^\ell ba^p$, where $j + k + \ell = p$ since $xyz = s = a^pba^p$. Also, $|y| > 0$ implies $k > 0$. Now consider the string $xyyz = a^ja^ka^ka^\ell ba^p = a^{p+k}ba^p$ since $j + k + \ell = p$. Note that $xyyz \notin A$ since it is not the same forwards and backwards because $k > 0$, which contradicts (i), so A is not a regular language.