Midterm Exam
CS 341-451: Foundations of Computer Science II — **Fall 2018, eLearning section**
Prof. Marvin K. Nakayama

Print family (or last) name: ______________________________

Print given (or first) name: ______________________________

I have read and understand all of the instructions below, and I will obey the University Policy on Academic Integrity.

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Signature and Date

- This exam has 9 pages in total, numbered 1 to 9. Make sure your exam has all the pages.
- Unless other arrangements have been made with the professor, the exam is to last 2.5 hours. and is to be given on Saturday, 10/20/18.
- This is a closed-book, closed-note exam. Electronic devices (e.g., cellphone, smart watch, calculator) are not allowed.
- For all problems, follow these instructions:
  1. Give only your answers in the spaces provided. I will only grade what you put in the answer space, and I will take off points for any scratch work in the answer space. Use the scratch-work area or the backs of the sheets to work out your answers before filling in the answer space.
  2. DFA stands for deterministic finite automaton; NFA stands for nondeterministic finite automaton; CFG stands for context-free grammar; PDA stands for pushdown automaton; TM stands for Turing machine.
  3. For any machines that you draw, you must include **all states and transitions**.
  4. For any proofs, be sure to provide a step-by-step argument, with justifications for every step. If you are asked to prove a result X, in your proof of X, you may use any other result Y without proving Y. However, make it clear what the other result Y is that you are using; e.g., write something like, "By the result that $A^{**} = A^*$, we know that ...."

Problem	1	2	3	4	5	6	7	Total
Points								

1. **[20 points]** For each of the following, circle TRUE if the statement is correct. Otherwise, circle FALSE

(a) TRUE FALSE — If A is a finite language, then A is context-free.

(b) TRUE FALSE — If $A \subseteq B$ and B is a regular language, then A is a regular language.

(c) TRUE FALSE — The class of context-free languages is closed under complementation.

(d) TRUE FALSE — Every nonregular language is context-free.

(e) TRUE FALSE — If A is a regular language, then A is finite.

(f) TRUE FALSE — The language $\{ a^n b^n \mid n \geq 0 \}$ has regular expression a^*b^*.

(g) TRUE FALSE — If A is a context-free language that is also non-regular, then A has a CFG in Chomsky normal form.

(h) TRUE FALSE — If $A \subseteq B$ and A is a context-free language, then B is a context-free language.

(i) TRUE FALSE — If A has an NFA and B is a finite language, then $A \cap \overline{B}$ is context-free.

(j) TRUE FALSE — If $M = (Q, \Sigma, \Gamma, \delta, q_0, q_{\text{accept}}, q_{\text{reject}})$ is a Turing machine and $w \in \Sigma^*$ is a string, the TM M either accepts or rejects w.

2. **[20 points]** Give short answers to each of the following parts. **Each answer should be at most a few sentences. Be sure to define any notation that you use.**

(a) Let $\Sigma = \{a, b\}$, and let A be the set of strings that have exactly two a's or at least two b's. Give a regular expression for A.

(b) Suppose that we are in the process of converting a CFG G with $\Sigma = \{a, b\}$ into Chomsky normal form. We have already applied some steps in the process, and we currently have the following CFG:

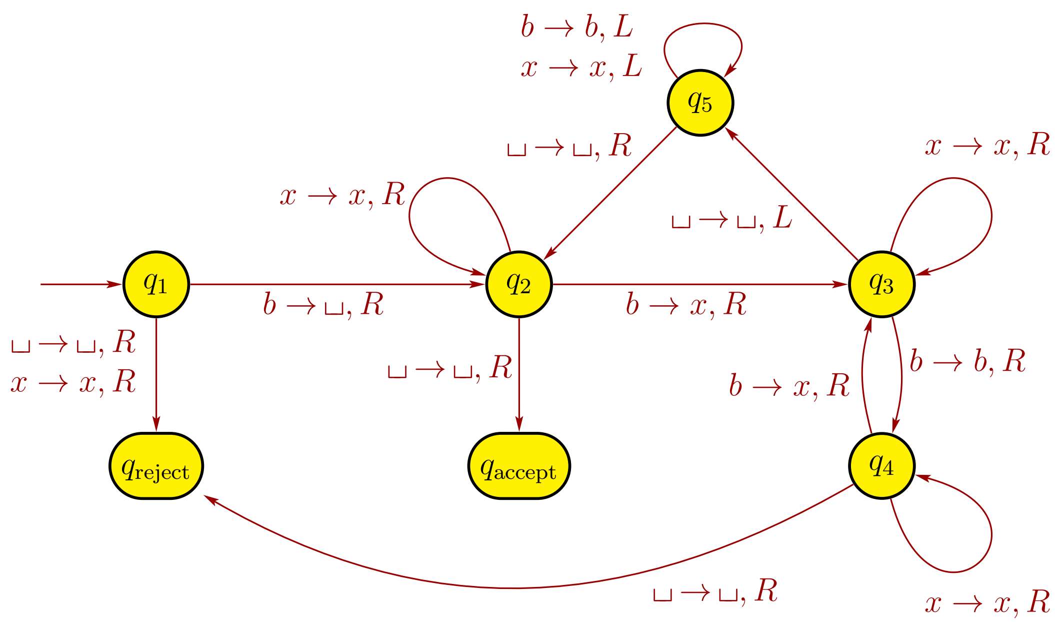
$$\begin{aligned} S_0 &\rightarrow S \\ S &\rightarrow bXSa \mid SaXbXS \mid \varepsilon \\ X &\rightarrow aSb \mid \varepsilon \end{aligned}$$

In the next step, we want to remove the ε-rule $X \rightarrow \varepsilon$. Give the CFG after carrying out just this one step.

(c) Let $\Sigma = \{0, 1\}$, and define $B = \{\, w \in \Sigma^* \mid w \text{ ends in } 010 \,\}$. Draw a DFA for B. You only need to draw the graph.

(d) Suppose that A_1 is a language defined by a CFG $G_1 = (V_1, \Sigma, R_1, S_1)$, and A_2 is a language defined by a CFG $G_2 = (V_2, \Sigma, R_2, S_2)$, where the alphabet Σ is the same for both languages and $V_1 \cap V_2 = \emptyset$. Let $A_3 = A_1 \cup A_2$. Give a CFG G_3 for A_3 in terms of G_1 and G_2. You do not have to prove the correctness of your CFG G_3, but do not give just an example.

3. **[10 points]** The Turing machine M below has input alphabet $\Sigma = \{b\}$ and tape alphabet $\Gamma = \{b, x, \sqcup\}$.



Give the sequence of configurations that M enters when started on the input string bb.

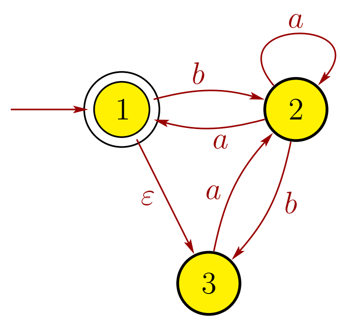
Scratch-work area

4. [**10 points**]

(a) Show by giving an example that, if M is an NFA that recognizes language C, swapping the accept and non-accept states in M does not necessarily yield a new NFA that recognizes $\overline{C}$. Explain your answer.

(b) Is the class of languages recognized by NFAs closed under complements? Explain your answer.

5. **[10 points]** Convert the following NFA into an equivalent DFA.



Answer:

Scratch-work area

6. [**15 points**] Consider the language

$$A = \{\, c^i a^j b^k \mid i, j, k \geq 0 \text{ and } i = j + k \,\}.$$

(a) Give a context-free grammar G that generates the language A. Be sure to specify G as a 4-tuple $G = (V, \Sigma, R, S)$.

(b) Give a PDA for the language A. You only need to draw the picture.

Scratch-work area

7. **[15 points]** Recall the pumping lemma for regular languages:

Theorem: If L is a regular language, then there is a number p (pumping length) where, if $s \in L$ with $|s| \geq p$, then s can be split into three pieces $s = xyz$ such that

(i) $xy^iz \in L$ for each $i \geq 0$,

(ii) $|y| > 0$, and

(iii) $|xy| \leq p$.

Let $\Sigma = \{a, b\}$, and consider the language $A = \{\, w \in \Sigma^* \mid w = w^{\mathcal{R}}, |w| \text{ is odd } \}$, where $w^{\mathcal{R}}$ denotes the reverse of w and $|w|$ denotes the length of w. Is A a regular or nonregular language? If A is regular, give a regular expression for A. If A is not regular, prove that it is a nonregular language.

Circle one: **Regular Language** **Nonregular Language**