Midterm Exam 1
CS 341: Foundations of Computer Science II — **Fall 2018, day section**
Prof. Marvin K. Nakayama

Print family (or last) name: _____

Print given (or first) name: _____

I have read and understand all of the instructions below, and I will obey the University Policy on Academic Integrity.

_____
Signature and Date

- This exam has 7 pages in total, numbered 1 to 7. Make sure your exam has all the pages.
- Note the number written on the upper right-hand corner of the first page. On the sign-up sheet being passed around, print your name next to this number.
- This exam will be 1 hour and 20 minutes in length.
- This is a closed-book, closed-note exam. Electronic devices (e.g., cellphone, smart watch, calculator) are not allowed.
- For all problems, follow these instructions:
  1. Give only your answers in the spaces provided. I will only grade what you put in the answer space, and I will take off points for any scratch work in the answer space. Use the scratch-work area or the backs of the exam sheets to work out your answers before filling in the answer space.
  2. DFA stands for deterministic finite automaton; NFA stands for nondeterministic finite automaton; PDA stands for push-down automaton; CFG stands for context-free grammar.
  3. For any proofs, be sure to provide a step-by-step argument, with justifications for every step. If you are asked to prove a result X, you may use in your proof of X any other result Y without proving Y. However, make it clear what the other result Y is that you are using; e.g., write something like, "By the result that $A^{**} = A^*$, we know that ...."

Problem	1	2	3	4	5	Total
Points						

1. **[20 points]** For each of the following, circle TRUE if the statement is correct. Otherwise, circle FALSE

(a) TRUE FALSE — If $A \subseteq B$ and A is a regular language, then B must be regular.

(b) TRUE FALSE — If A is regular, then A must be finite.

(c) TRUE FALSE — Every context-free language is also regular.

(d) TRUE FALSE — The class of regular languages is closed under intersection.

(e) TRUE FALSE — A regular expression for $A = \{ 0^n 1^n 0^n \mid n \geq 0 \}$ is $0^*1^*0^*$.

(f) TRUE FALSE — If A has a regular expression, then A must have a PDA.

(g) TRUE FALSE — If a language A has a PDA, then A must have a context-free grammar in Chomsky normal form.

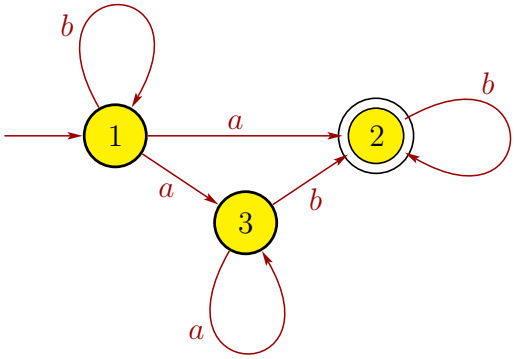
(h) TRUE FALSE — For an NFA $N = (Q, \Sigma, \delta, q_0, F)$, its transition function has the form $\delta : Q \times \Sigma_\varepsilon \to Q$.

(i) TRUE FALSE — If a language A is nonregular, then A has an NFA.

(j) TRUE FALSE — If $A \subseteq B$ and B is a regular language, then A must be regular.

2. **[20 points]** Give short answers to each of the following parts. Each answer should be at most a few sentences. Be sure to define any notation that you use.

   (a) Give a regular expression for the language recognized by the NFA below.



   (b) Suppose A is generated by a context-free grammar $G = (V, \Sigma, R, S)$. Give a context-free grammar G' for A^* in terms of G. You do not have to prove the correctness of your CFG G', but do not just give an example.

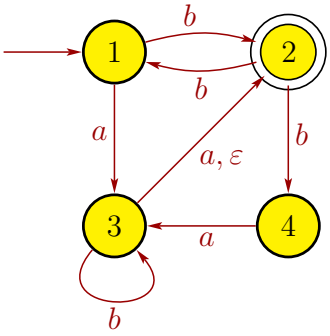
(c) Let $M_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$ be a DFA with language A_1, and $M_2 = (Q_2, \Sigma, \delta_2, q_2, F_2)$ be a DFA with language A_2. Consider the language $A = A_1 \cap A_2$. Give a DFA M_3 for A in terms of M_1 and M_2. Your DFA M_3 must be completely general. Do not prove the correctness of your DFA M_3, but do not just give an example.

(d) Suppose that we are in the process of converting a CFG G with $\Sigma = \{0, 1\}$ into Chomsky normal form. We have already applied some steps in the process, and we currently have the following CFG:

$$\begin{aligned} S_0 &\to S \\ S &\to A1AS \mid 01A \mid \varepsilon \\ A &\to 1S0 \mid \varepsilon \end{aligned}$$

In the next step, we want to remove the ε-rule $A \to \varepsilon$. Give the CFG after carrying out just this one step.

3. **[20 points]** Let N be the following NFA with $\Sigma = \{a, b\}$, and let $C = L(N)$.



Give a DFA for C.

Scratch-work area

4. **[25 points]** Consider the language

$$L = \{\, c^i a^j b^k \mid i, j, k \geq 0, \text{ and } i = j \text{ or } j = k \,\}.$$

(a) Give a context-free grammar G for L. Be sure to specify G as a 4-tuple $G = (V, \Sigma, R, S)$.

(b) Give a PDA for L. You only need to draw the graph.

Scratch-work area

5. **[15 points]** Recall the pumping lemma for regular languages:

   Theorem: If L is a regular language, then there exists a pumping length p where, if $s \in L$ with $|s| \geq p$, then s can be split into three pieces $s = xyz$ such that (i) $xy^iz \in L$ for each $i \geq 0$, (ii) $|y| \geq 1$, and (iii) $|xy| \leq p$.

   Let $A = \{ c^i a^j b^k \mid i, j, k \geq 0, \text{ and } i = j \text{ or } j = k \}$. Is A a regular or nonregular language? If A is regular, give a regular expression for A. If A is not regular, prove that it is a nonregular language.

   Circle one: **Regular Language** **Nonregular Language**