Midterm Exam
CS 341-452: Foundations of Computer Science II — **Spring 2018, eLearning section**
Prof. Marvin K. Nakayama

Print family (or last) name: ____

Print given (or first) name: ____

I have read and understand all of the instructions below, and I will obey the University Policy on Academic Integrity.

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Signature and Date

- This exam has 9 pages in total, numbered 1 to 9. Make sure your exam has all the pages.
- Unless other arrangements have been made with the professor, the exam is to last 2.5 hours. and is to be given on Saturday, March 3, 2018.
- This is a closed-book, closed-note exam. Electronic devices (e.g., calculators, cellphones, smart watches) are not allowed. If you take out an electronic device or notes during the exam, your exam will be taken away immediately and you will be reported to the Dean of Students.
- For all problems, follow these instructions:
  1. Give only your answers in the spaces provided. I will only grade what you put in the answer space, and I will take off points for any scratch work in the answer space. Use the scratch-work area or the backs of the sheets to work out your answers before filling in the answer space.
  2. DFA stands for deterministic finite automaton; NFA stands for nondeterministic finite automaton; CFG stands for context-free grammar; PDA stands for pushdown automaton; TM stands for Turing machine.
  3. For any proofs, be sure to provide a step-by-step argument, with justifications for every step. If you are asked to prove a result X, in your proof of X, you may use any other result Y without proving Y. However, make it clear what the other result Y is that you are using; e.g., write something like, "By the result that $A^{**} = A^*$, we know that ...."

Problem	1	2	3	4	5	6	7	Total
Points								

1. **[20 points]** For each of the following, circle TRUE if the statement is correct. Otherwise, circle FALSE

(a) TRUE FALSE — If $A \subseteq B$ and A is a regular language, then B must be regular.

(b) TRUE FALSE — If A is a regular language, then A must be finite.

(c) TRUE FALSE — Every context-free language is also regular.

(d) TRUE FALSE — The class of regular languages is closed under intersection.

(e) TRUE FALSE — A regular expression for $A = \{ 0^n1^n \mid n \geq 0 \}$ is 0^*1^*.

(f) TRUE FALSE — If A has an NFA, then A is nonregular.

(g) TRUE FALSE — If a language A has a DFA, then A must have a PDA.

(h) TRUE FALSE — If a language A has a PDA, then A must have a context-free grammar in Chomsky normal form.

(i) TRUE FALSE — The transition function of an NFA is $\delta : Q \times \Sigma_\varepsilon \to Q$.

(j) TRUE FALSE — If $A \subseteq B$ and B is a regular language, then A must be regular.

2. **[20 points]** Give short answers to each of the following parts. **Each answer should be at most a few sentences. Be sure to define any notation that you use.**

   (a) Let $\Sigma = \{a, b\}$, and we say a string $w \in \Sigma^*$ contains a *double letter* if aa or bb is a substring of w. Define the language

   $$A = \{\, w \in \Sigma^* \mid w \text{ contains exactly one double letter} \,\}.$$

   For example, $baaba$ has exactly one double letter, but $baaaba$ has two double letters. Give a regular expression for A.

   (b) Suppose that we are in the process of converting a CFG G with $\Sigma = \{0, 1\}$ into Chomsky normal form. We have already applied some steps in the process, and we currently have the following CFG:

   $$\begin{aligned} S_0 &\rightarrow S \\ S &\rightarrow A1SA \mid A0 \mid \varepsilon \\ A &\rightarrow 0S0 \mid \varepsilon \end{aligned}$$

   In the next step, we want to remove the ε-rule $A \rightarrow \varepsilon$. Give the CFG after carrying out just this one step.

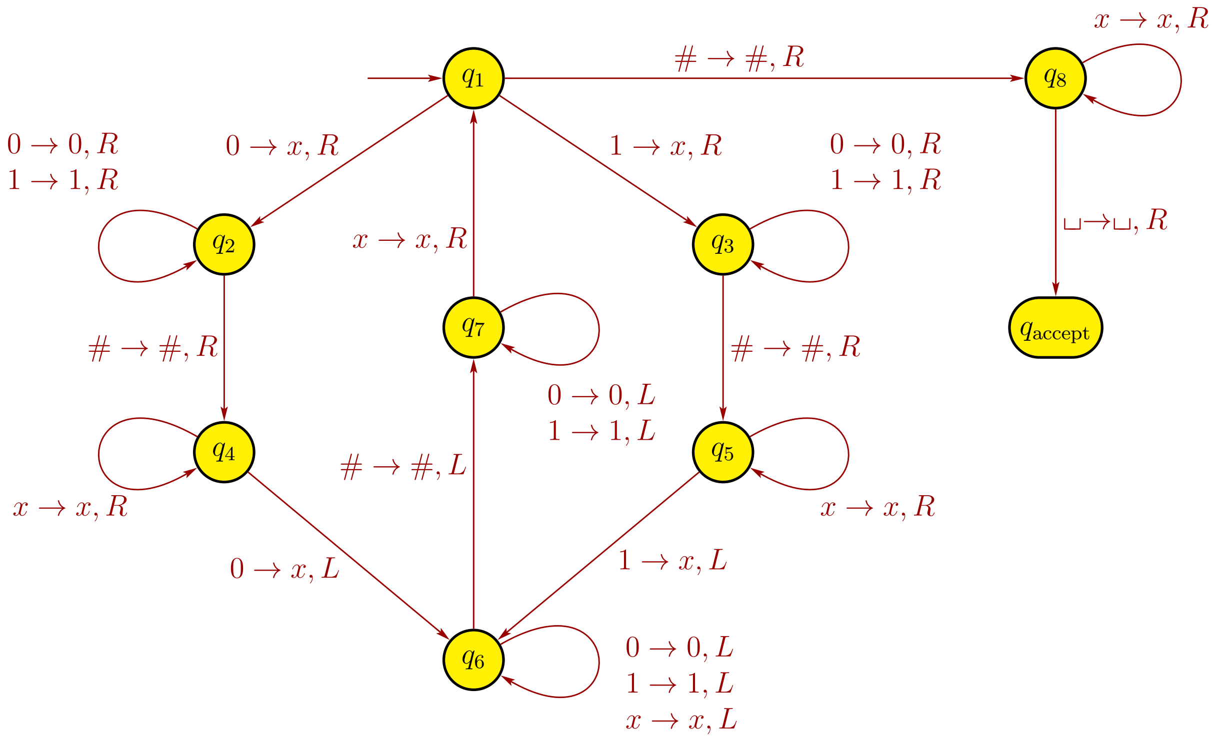
(c) For $\Sigma = \{a, b\}$, give a DFA (drawing only) that recognizes the language

$$C = \{\, w \in \Sigma^* \mid |w| \geq 2, \text{ second-to-last symbol of } w \text{ is } a \,\}.$$

If string $w = w_1 w_2 \cdots w_n$ with $n \geq 2$ and each $w_i \in \Sigma$, then the second-to-last symbol of w is w_{n-1}.

(d) Suppose that A_1 is a language defined by a CFG $G_1 = (V_1, \Sigma, R_1, S_1)$, and A_2 is a language defined by a CFG $G_2 = (V_2, \Sigma, R_2, S_2)$, where the alphabet Σ is the same for both languages and $V_1 \cap V_2 = \emptyset$. Let $A_3 = A_1 \circ A_2$. Give a CFG G_3 for A_3 in terms of G_1 and G_2. You do not have to prove the correctness of your CFG G_3, but do not give just an example.

3. [**10 points**] Consider the below Turing machine $M = (Q, \Sigma, \Gamma, \delta, q_1, q_{\text{accept}}, q_{\text{reject}})$ with $Q = \{q_1, \dots, q_8, q_{\text{accept}}, q_{\text{reject}}\}$, $\Sigma = \{0, 1, \#\}$, $\Gamma = \{0, 1, \#, x, \sqcup\}$, and transitions below.

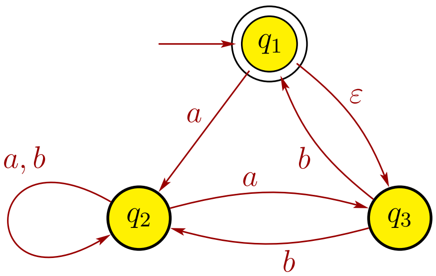


To simplify the figure, we don't show the reject state q_{reject} or the transitions going to the reject state. Those transitions occur implicitly whenever a state lacks an outgoing transition for a particular symbol. For example, because in state q_5 no outgoing arrow with a # is present, if a # occurs under the head when the machine is in state q_5, it goes to state q_{reject}. For completeness, we say that in each of these transitions to the reject state, the head writes the same symbol as is read and moves right.

Give the sequence of configurations that M enters when started on the input string 0#0.

Scratch-work area

4. **[10 points]** Let N be the following NFA with $\Sigma = \{a, b\}$, and let $C = L(N)$.



Give a DFA for C.

Answer:

Scratch-work area

5. **[20 points]** Let $\Sigma = \{a, b, c\}$, and consider the language $A = \{\, a^n b^k c^n \mid n, k \geq 0 \,\}$.

(a) Give a CFG G for A. Be sure to specify G as a 4-tuple $G = (V, \Sigma, R, S)$.

(b) Give a PDA for A. You only need to give the drawing.

Scratch-work area

Each of the following problems requires you to prove a result. Unless stated otherwise, in your proofs, you can apply any theorems or results that we went over in class (lectures, homeworks) without proving them, except for the result you are asked to prove in the problem. When citing a theorem or result, make sure that you give enough details so that it is clear what theorem or result you are using (e.g., say something like, "By the result that says $A^{**} = A^*$, we can show that ....")

6. **[10 points]** Show that the class of context-free languages is not closed under complementation. Be sure to explain your answer. You must show that any specific languages you claim are context-free are indeed context-free, e.g., by providing a CFG. [Hint: consider languages similar to that in problem 5, and use DeMorgan's law: $A \cap B = \overline{\overline{A} \cup \overline{B}}$.]

7. **[10 points]** Recall the pumping lemma for regular languages:

Theorem: If L is a regular language, then there is a number p (pumping length) where, if $s \in L$ with $|s| \geq p$, then s can be split into three pieces $s = xyz$ such that

(i) $xy^iz \in L$ for each $i \geq 0$,

(ii) $|y| > 0$, and

(iii) $|xy| \leq p$.

Let $\Sigma = \{a, b\}$, and consider the language $A = \{\, w \in \Sigma^* \mid w = w^{\mathcal{R}}, |w| \text{ is even} \,\}$, where $w^{\mathcal{R}}$ denotes the reverse of string w and $|w|$ denotes the length of w. Prove that A is not a regular language.