Midterm Exam
CS 341-452: Foundations of Computer Science II — **Spring 2019, eLearning section**
Prof. Marvin K. Nakayama

Print family (or last) name: ____

Print given (or first) name: ____

I have read and understand all of the instructions below, and I will obey the University Policy on Academic Integrity.

____
Signature and Date

- This exam has 9 pages in total, numbered 1 to 9. Make sure your exam has all the pages.
- Unless other arrangements have been made with the professor, the exam is to last 2.5 hours. and is to be given on Saturday, March 9, 2019.
- This is a closed-book, closed-note exam. Electronic devices (e.g., cellphone, smart watch, calculator) are not allowed.
- For all problems, follow these instructions:
  1. Give only your answers in the spaces provided. I will only grade what you put in the answer space, and I will take off points for any scratch work in the answer space. Use the scratch-work area or the backs of the sheets to work out your answers before filling in the answer space.
  2. DFA stands for deterministic finite automaton; NFA stands for nondeterministic finite automaton; CFG stands for context-free grammar; PDA stands for pushdown automaton; TM stands for Turing machine.
  3. For any state machines that you draw, you must include **all states and transitions**, unless otherwise specified.
  4. For any proofs, be sure to provide a step-by-step argument, with justifications for every step. If you are asked to prove a result X, in your proof of X, you may use any other result Y without proving Y. However, make it clear what the other result Y is that you are using; e.g., write something like, "By the result that $A^{**} = A^*$, we know that ...."

Problem	1	2	3	4	5	6	7	Total
Points								

1. **[20 points]** For each of the following, circle TRUE if the statement is correct. Otherwise, circle FALSE.

(a) TRUE FALSE — If A is a context-free language that is infinite, then A has a CFG in Chomsky normal form.

(b) TRUE FALSE — If A is a nonregular language and B is a finite language, then $A \cap B$ must be regular.

(c) TRUE FALSE — If $M = (Q, \Sigma, \Gamma, \delta, q_0, q_{\text{accept}}, q_{\text{reject}})$ is a Turing machine and $w \in \Sigma^*$ is a string, the TM M either accepts or rejects w.

(d) TRUE FALSE — If $A \subseteq B$ and A is a regular language, then B is a context-free language.

(e) TRUE FALSE — The class of context-free languages is closed under concatenation.

(f) TRUE FALSE — Every nonregular language is context-free.

(g) TRUE FALSE — If A is a context-free language, then A is finite.

(h) TRUE FALSE — If A is a finite language, then $\overline{A}$ is context-free.

(i) TRUE FALSE — If $A \subseteq B$ and B is a regular language, then A is a context-free language.

(j) TRUE FALSE — The language $\{ a^n b^n a^n \mid n \geq 0 \}$ has regular expression $a^* b^* a^*$.

2. **[20 points]** Give short answers to each of the following parts. **Each answer should be at most a few sentences. Be sure to define any notation that you use.**

   (a) Let $\Sigma = \{a, b\}$, and let $A = \{\, w \in \Sigma^* \mid w$ contains at least three b's or exactly two a's$\,\}$. Give a regular expression for A.

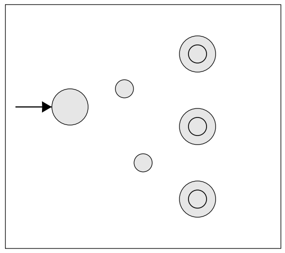
   (b) Consider the following CFG $G = (V, \Sigma, R, S)$, with $V = \{S, X, Y\}$, $\Sigma = \{a, b\}$, start variable S, and rules R as follows:

$$
\begin{aligned}
S &\rightarrow YY \mid ba \mid \varepsilon \\
X &\rightarrow YS \mid b \mid \varepsilon \\
Y &\rightarrow XY \mid X \mid aX
\end{aligned}
$$

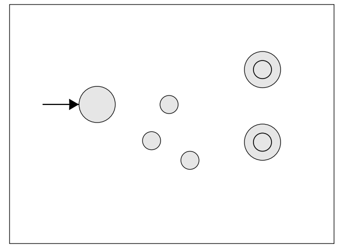
   Note that G is not in Chomsky normal form. List all of the rules in G that violate Chomsky normal form. Explain your answer.

(c) Suppose that language A_1 is recognized by NFA N_1 below, and language A_2 is recognized by NFA N_2 below. Note that the transitions are not drawn in N_1 and N_2. Draw a picture of an NFA for $A_1 \circ A_2$.

N_1

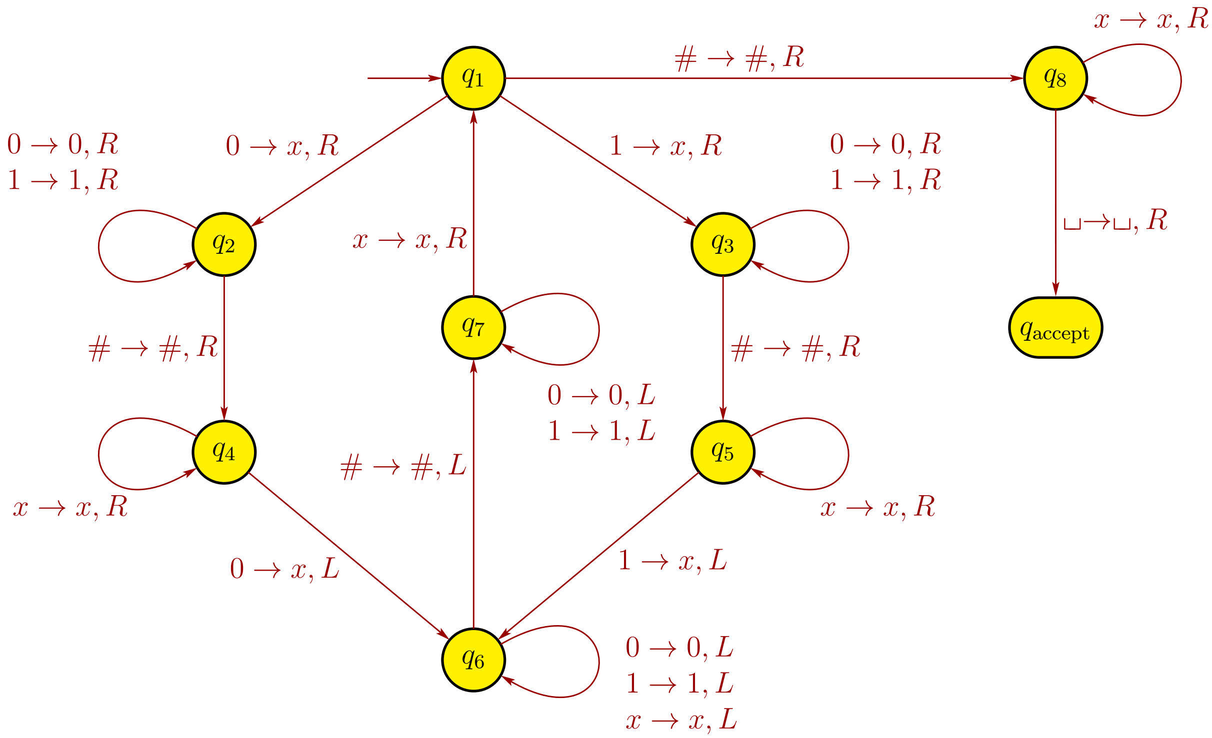


N_2



(d) Suppose that A_1 is a language defined by a CFG $G_1 = (V_1, \Sigma, R_1, S_1)$, and A_2 is a language defined by a CFG $G_2 = (V_2, \Sigma, R_2, S_2)$, where the alphabet Σ is the same for both languages and $V_1 \cap V_2 = \emptyset$. Let $A_3 = A_1 \cup A_2$. Give a CFG G_3 for A_3 in terms of G_1 and G_2. You do not have to prove the correctness of your CFG G_3, but do not give just an example.

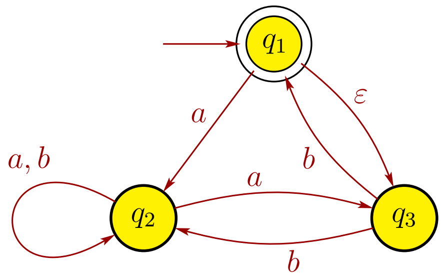
3. **[10 points]** Consider the below Turing machine $M = (Q, \Sigma, \Gamma, \delta, q_1, q_{\text{accept}}, q_{\text{reject}})$ with $Q = \{q_1, \ldots, q_8, q_{\text{accept}}, q_{\text{reject}}\}$, $\Sigma = \{0, 1, \#\}$, $\Gamma = \{0, 1, \#, x, \sqcup\}$, and transitions below.



To simplify the figure, we don't show the reject state q_{reject} or the transitions going to the reject state. Those transitions occur implicitly whenever a state lacks an outgoing transition for a particular symbol. For example, because in state q_5 no outgoing arrow with a # is present, if a # occurs under the head when the machine is in state q_5, it goes to state q_{reject}. For completeness, we say that in each of these transitions to the reject state, the head writes the same symbol as is read and moves right.

Give the sequence of configurations that M enters when started on the input string 10#1100.

4. **[10 points]** Let N be the following NFA with $\Sigma = \{a, b\}$, and let $C = L(N)$.



Give a DFA for C.

Answer:

Scratch-work area

5. **[20 points]** Let $\Sigma = \{a, b, c\}$, and consider the language $A = \{ b^{2n} c^k a^{3n} \mid n \geq 0, k \geq 0 \}$.

(a) Give a CFG G for A. Be sure to specify G as a 4-tuple $G = (V, \Sigma, R, S)$.

(b) Give a PDA for A. You only need to give the drawing.

Scratch-work area

Each of the following problems requires you to prove a result. Unless stated otherwise, in your proofs, you can apply any theorems or results that we went over in class (lectures, homeworks) without proving them, except for the result you are asked to prove in the problem. When citing a theorem or result, make sure that you give enough details so that it is clear what theorem or result you are using (e.g., say something like, "By the result that says $A^{**} = A^*$, we can show that ....")

6. **[10 points]** We say that a DFA M for a language A is *minimal* if there does not exist another DFA M' for A such that M' has strictly fewer states than M. Suppose that $M = (Q, \Sigma, \delta, q_0, F)$ is a minimal DFA for A. Using M, we construct a DFA $\overline{M}$ for the complement $\overline{A}$ as $\overline{M} = (Q, \Sigma, \delta, q_0, Q - F)$. Prove that $\overline{M}$ is a minimal DFA for $\overline{A}$.

7. **[10 points]** Recall the pumping lemma for regular languages:

Theorem: If L is a regular language, then there is a number p (pumping length) where, if $s \in L$ with $|s| \geq p$, then s can be split into three pieces, $s = xyz$, satisfying the conditions

(i) $xy^iz \in L$ for each $i \geq 0$,

(ii) $|y| > 0$, and

(iii) $|xy| \leq p$.

Consider the language $A = \{ b^{2n}c^ka^{3n} \mid n \geq 0, k \geq 0 \}$. Is A a regular or nonregular language? If A is regular, give a regular expression for A. If A is not regular, prove that it is a nonregular language.

Circle one: **Regular Language** **Nonregular Language**