Midterm Exam 1
CS 341: Foundations of Computer Science II — **Fall 2020, hybrid section**
Prof. Marvin K. Nakayama

Print family (or last) name: ____

Print given (or first) name: ____

I have read and understand all of the instructions below, and I will obey the University Policy on Academic Integrity.

____
Signature and Date

- This exam has 7 pages in total, numbered 1 to 7. Make sure your exam has all the pages.
- Note the number written on the upper right-hand corner of the first page. On the sign-up sheet being passed around, print your name next to this number.
- This exam will be 1 hour and 20 minutes in length.
- This is a closed-book, closed-note exam. Electronic devices (e.g., cellphone, smart watch, calculator) are not allowed.
- For all problems, follow these instructions:
  1. Give only your answers in the spaces provided. I will only grade what you put in the answer space, and I will take off points for any scratch work in the answer space. Use the scratch-work area or the backs of the exam sheets to work out your answers before filling in the answer space.
  2. DFA stands for deterministic finite automaton; NFA stands for nondeterministic finite automaton; PDA stands for push-down automaton; CFG stands for context-free grammar.
  3. For any state machines that you draw, you must include **all states and transitions**.
  4. For any proofs, be sure to provide a step-by-step argument, with justifications for every step. If you are asked to prove a result X, you may use in your proof of X any other result Y without proving Y. However, make it clear what the other result Y is that you are using; e.g., write something like, "By the result that $A^{**} = A^*$, we know that . . . ."

Problem	1	2	3	4	5	Total
Points						

1. **[10 points]** For each of the following, circle TRUE if the statement is correct. Otherwise, circle FALSE

(a) TRUE FALSE — If a language A is nonregular, then A has an NFA.

(b) TRUE FALSE — If $A \subseteq B$ and B is a regular language, then A must be regular.

(c) TRUE FALSE — If $A \subseteq B$ and A is a regular language, then B must be regular.

(d) TRUE FALSE — If a language A is regular, then A must be finite.

(e) TRUE FALSE — If A is a context-free language, then A must also be regular.

(f) TRUE FALSE — If A is a context-free language, then $\overline{A}$ must also be context-free.

(g) TRUE FALSE — A regular expression for $A = \{ 0^n 1^n 0^n \mid n \geq 0 \}$ is $0^* 1^* 0^*$.

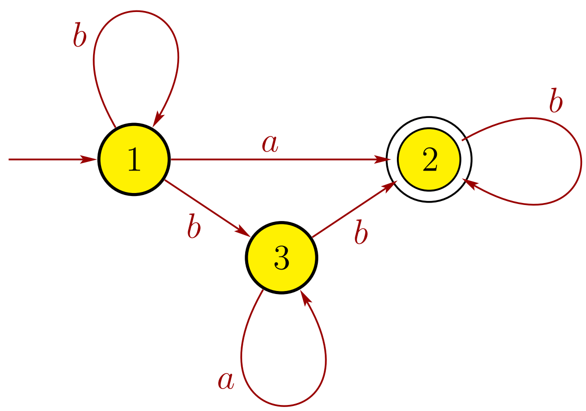
(h) TRUE FALSE — If a language A has a regular expression, then A must have a PDA.

(i) TRUE FALSE — If a language A has a PDA, then A must have a context-free grammar in Chomsky normal form.

(j) TRUE FALSE — For an NFA $N = (Q, \Sigma, \delta, q_0, F)$, its transition function has the form $\delta : Q \times \Sigma_\varepsilon \to Q$.

2. [**40 points**] Give short answers to each of the following parts. Each answer should be at most a few sentences. Be sure to define any notation that you use.

(a) Give a regular expression for the language recognized by the NFA below.



(b) Suppose a language A_1 is generated by a context-free grammar $G_1 = (V_1, \Sigma, R_1, S_1)$, and a language A_2 is generated by a context-free grammar $G_2 = (V_2, \Sigma, R_2, S_2)$, where for $i = 1, 2$, we have that V_i is the set of variables in G_i, R_i is the set of rules in G_i, and S_i is the start variable of G_i. The two grammars shard the same set of terminals Σ, and $V_1 \cap V_2 = \emptyset$. Give a context-free grammar G_3 for $A_1 \circ A_2$ in terms of G_1 and G_2. You do not have to prove the correctness of your CFG G_3, but do not just give an example.

(c) Let $M_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$ be a DFA with language A_1, and $M_2 = (Q_2, \Sigma, \delta_2, q_2, F_2)$ be a DFA with language A_2. Consider the language $A = A_1 \cap A_2$. Give a DFA M_3 for A in terms of M_1 and M_2. Your DFA M_3 must be completely general. Do not prove the correctness of your DFA M_3, but do not just give an example.

(d) Suppose that we are in the process of converting a CFG G with $\Sigma = \{0, 1\}$ into Chomsky normal form. We have already applied some steps in the process, and we currently have the following CFG:

$$\begin{aligned} S_0 &\rightarrow S \\ S &\rightarrow 0A1AS \mid 0A1 \mid \varepsilon \\ A &\rightarrow 10A1 \mid \varepsilon \end{aligned}$$

In the next step, we want to remove the ε-rule $A \rightarrow \varepsilon$. Give the CFG after carrying out just this one step.

3. **[20 points]** For $\Sigma = \{a, b\}$, consider the language

$$C = \{\, w \in \Sigma^* \mid |w| \geq 2, \text{ second-to-last symbol of } w \text{ is } a \,\}.$$

If string $w = w_1 w_2 \cdots w_n$ with $n \geq 2$ and each $w_i \in \Sigma$, then the second-to-last symbol of w is w_{n-1}.

(a) Give a 5-tuple description for a DFA for C. Be sure to explicit define each part of the 5-tuple for your DFA for C.

(b) Give a regular expression for C.

4. [**15 points**] Consider the language

$$D = \{\, c^i a^j b^k \mid i, j, k \geq 0, \text{ and } i = j + k \,\}.$$

Give a context-free grammar G for D. Be sure to specify G as a 4-tuple $G = (V, \Sigma, R, S)$.

5. **[15 points]** Recall the pumping lemma for regular languages:

Theorem: If L is a regular language, then there exists a pumping length p where, if $s \in L$ with $|s| \geq p$, then s can be split into three pieces $s = xyz$ such that (i) $xy^iz \in L$ for each $i \geq 0$, (ii) $|y| \geq 1$, and (iii) $|xy| \leq p$.

Let $D = \{\, c^i a^j b^k \mid i, j, k \geq 0, \text{ and } i = j + k \,\}$. Is D a regular or nonregular language? If D is regular, give a regular expression for D. If D is not regular, prove that it is a nonregular language.

Circle one: **Regular Language** **Nonregular Language**