Midterm Exam 1
CS 341: Foundations of Computer Science II — **Spring 2020, hybrid section**
Prof. Marvin K. Nakayama

Print family (or last) name: _____

Print given (or first) name: _____

I have read and understand all of the instructions below, and I will obey the University Policy on Academic Integrity.

_____
Signature and Date

- This exam has 7 pages in total, numbered 1 to 7. Make sure your exam has all the pages.
- Note the number written on the upper right-hand corner of the first page. On the sign-up sheet being passed around, print your name next to this number.
- This exam will be 1 hour and 20 minutes in length.
- This is a closed-book, closed-note exam. Electronic devices (e.g., cellphone, smart watch, calculator) are not allowed.
- For all problems, follow these instructions:
    1. Give only your answers in the spaces provided. I will only grade what you put in the answer space, and I will take off points for any scratch work in the answer space. Use the scratch-work area or the backs of the exam sheets to work out your answers before filling in the answer space.
    2. DFA stands for deterministic finite automaton; NFA stands for nondeterministic finite automaton; PDA stands for push-down automaton; CFG stands for context-free grammar.
    3. For any state machines that you draw, you must include **all states and transitions**.
    4. For any proofs, be sure to provide a step-by-step argument, with justifications for every step. If you are asked to prove a result X, you may use in your proof of X any other result Y without proving Y. However, make it clear what the other result Y is that you are using; e.g., write something like, "By the result that $A^{**} = A^*$, we know that ...."

Problem	1	2	3	4	5	Total
Points						

1. **[20 points]** For each of the following, circle TRUE if the statement is correct. Otherwise, circle FALSE

(a) TRUE FALSE — Every context-free language is also regular.

(b) TRUE FALSE — The class of regular languages is closed under intersection.

(c) TRUE FALSE — A regular expression for $A = \{ 0^n1^n0^n \mid n \geq 0 \}$ is $0^*1^*0^*$.

(d) TRUE FALSE — If A has a regular expression, then A must have a PDA.

(e) TRUE FALSE — If a language A has a PDA, then A must have a context-free grammar in Chomsky normal form.

(f) TRUE FALSE — For an NFA $N = (Q, \Sigma, \delta, q_0, F)$, its transition function has the form $\delta : Q \times \Sigma_\varepsilon \to Q$.

(g) TRUE FALSE — If a language A is nonregular, then A has an NFA.

(h) TRUE FALSE — If $A \subseteq B$ and B is a context-free language, then A must be context-free.

(i) TRUE FALSE — If $A \subseteq B$ and A is a regular language, then B must be regular.

(j) TRUE FALSE — If A is regular, then A must be finite.

2. **[20 points]** Give short answers to each of the following parts. Each answer should be at most a few sentences. Be sure to define any notation that you use.

   (a) Let $\Sigma = \{0, 1\}$. For any string $w \in \Sigma^*$, let $n_0(w)$ denote the number of 0's in w, and let $n_1(w)$ denote the number of 1's in w. Define the language $A = \{ w \in \Sigma^* \mid n_0(w) \leq 1, n_1(w) \geq 2 \}$. Give a DFA for A.

   Answer:

   (b) Give a regular expression for the language $A = \{ w \in \Sigma^* \mid n_0(w) \leq 1, n_1(w) \geq 2 \}$, where $\Sigma = \{0, 1\}$.

   Answer: _____

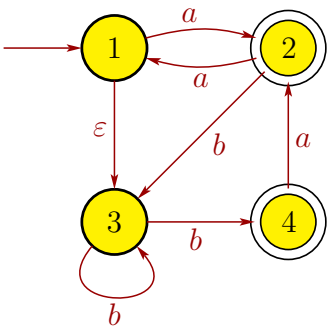
(c) Suppose a language A is generated by a context-free grammar $G = (V, \Sigma, R, S)$. Give a context-free grammar G' for A^* in terms of G. You do not have to prove the correctness of your CFG G', but do not just give an example.

(d) Suppose that we are in the process of converting a CFG G with $\Sigma = \{0, 1\}$ into Chomsky normal form. We have already applied some steps in the process, and we currently have the following CFG:

$$\begin{aligned} S_0 &\to S \\ S &\to 1SA0A \mid 0AS1S \mid \varepsilon \\ A &\to 10S1 \mid \varepsilon \end{aligned}$$

In the next step, we want to remove the ε-rule $A \to \varepsilon$. Give the CFG after carrying out just this one step.

3. **[15 points]** Let N be the following NFA with $\Sigma = \{a, b\}$, and let $C = L(N)$.



Give a DFA for C.

Scratch-work area

4. **[30 points]** Consider the alphabet $\Sigma = \{a, b\}$ and the language

$$L = \{\, b^i a^j \mid i \geq j \geq 0 \,\}.$$

(a) Give a context-free grammar G for L. Be sure to specify G as a 4-tuple $G = (V, \Sigma, R, S)$.

(b) Give a PDA for L. You only need to draw the graph.

Scratch-work area

5. **[15 points]** Recall the pumping lemma for regular languages:

   Theorem: If L is a regular language, then there exists a pumping length p where, if $s \in L$ with $|s| \geq p$, then s can be split into three pieces $s = xyz$ such that (i) $xy^iz \in L$ for each $i \geq 0$, (ii) $|y| \geq 1$, and (iii) $|xy| \leq p$.

   Let $A = \{ b^ia^j \mid i \geq j \geq 0\}$. Is A a regular or nonregular language? If A is regular, give a regular expression for A. If A is not regular, prove that it is a nonregular language.

   Circle one: **Regular Language** **Nonregular Language**