Midterm Exam
CS 341-451: Foundations of Computer Science II — **Fall 2021, eLearning (online) section**
Prof. Marvin K. Nakayama

Print family (or last) name: ____

Print given (or first) name: ____

I have read and understand all of the instructions below, and I will obey the University Policy on Academic Integrity.

____
Signature and Date

- This exam has 11 pages in total, numbered 1 to 11. Make sure your exam has all the pages.
- Unless other arrangements have been made with the professor, the exam is to last 2.5 hours. and is to be given on Saturday, 10/30/2021.
- This is a closed-book, closed-note exam. Electronic devices (e.g., cellphone, smart watch, calculator) are not allowed.
- For all problems, follow these instructions:
  1. Give only your answers in the spaces provided. I will only grade what you put in the answer space, and I will take off points for any scratch work in the answer space. Use the scratch-work area or the backs of the sheets to work out your answers before filling in the answer space.
  2. DFA stands for deterministic finite automaton; NFA stands for nondeterministic finite automaton; CFG stands for context-free grammar; PDA stands for pushdown automaton; TM stands for Turing machine.
  3. For any state machines that you draw, you must include **all states and transitions**, unless otherwise specified.
  4. For any proofs, be sure to provide a step-by-step argument, with justifications for every step. If you are asked to prove a result X, in your proof of X, you may use any other result Y without proving Y. However, make it clear what the other result Y is that you are using; e.g., write something like, “By the result that $A^{**} = A^*$, we know that ....”

Problem	1	2	3	4	5	6	7	8	Total
Points									

1. **[10 points]** For each of the following, circle TRUE if the statement is correct. Otherwise, circle FALSE

(a) TRUE FALSE — If a language A has an NFA, then A is context-free.

(b) TRUE FALSE — If A and B are nonregular languages, then $A \cup B$ must be nonregular.

(c) TRUE FALSE — If A and B are non-context-free languages, then $A \cap B$ must be non-context-free.

(d) TRUE FALSE — If A is a regular language, then A must be finite.

(e) TRUE FALSE — The class of Turing-decidable languages is closed under complementation.

(f) TRUE FALSE — If A and B are context-free languages, then $A \cap B$ must also be context-free.

(g) TRUE FALSE — A context-free grammar for $A = \{ 0^n 1^n 0^n \mid n \geq 0 \}$ is $G = (V, \Sigma, R, S)$, with $V = \{S\}$, $\Sigma = \{0, 1\}$, start variable S, and rules $R = \{ \ S \to 0S1S0S \mid \varepsilon \ \}$.

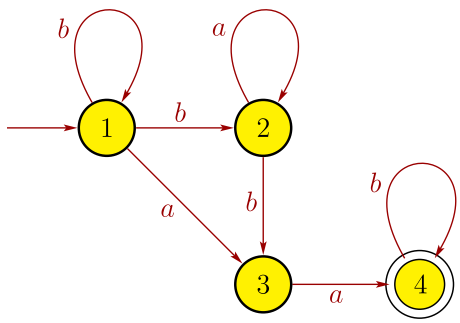
(h) TRUE FALSE — The class of Turing-decidable languages is a subset of the class of Turing-recognizable languages.

(i) TRUE FALSE — If a language A is nonregular, then A must have a context-free grammar in Chomsky normal form.

(j) TRUE FALSE — The class of languages having regular expressions is closed under concatenation.

2. **[20 points]** Give short answers to each of the following parts. Each answer should be at most a few sentences. Be sure to define any notation that you use.

(a) Give a regular expression for the language recognized by the NFA below.



(b) Suppose a language A_1 is generated by a context-free grammar $G_1 = (V_1, \Sigma, R_1, S_1)$, and a language A_2 is generated by a context-free $G_2 = (V_2, \Sigma, R_2, S_2)$, where for $i = 1, 2$, we have that V_i is the set of variables in G_i, R_i is the set of rules in G_i, and S_i is the start variable of G_i. The two grammars share the same set Σ of terminals, and $V_1 \cap V_2 = \emptyset$. Give a context-free grammar G_3 for $A_2 \circ A_1$ in terms of G_1 and G_2. You do not have to prove the correctness of your CFG G_3, but do not just give an example.

(c) Suppose that we are in the process of converting a CFG G with $\Sigma = \{0, 1\}$ into Chomsky normal form. We have already applied some steps in the process, and we currently have the following CFG with variables $V = \{S_0, S, A\}$ and start variable S_0:

$$\begin{array}{rcl} S_0 & \to & S \\ S & \to & ASA0A \mid \varepsilon \\ A & \to & 0SA \mid 0SA1S01A \mid \varepsilon \end{array}$$

In the next step, we want to remove the ε-rule $S \to \varepsilon$. Give the CFG after carrying out just this one step.

(d) Let $M_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$ be a DFA with language A_1, and $M_2 = (Q_2, \Sigma, \delta_2, q_2, F_2)$ be a DFA with language A_2. Consider the language $A = A_1 \cup A_2$. Give a DFA M_3 for A in terms of M_1 and M_2. Your DFA M_3 must be completely general. Do not prove the correctness of your DFA M_3, but do not just give an example.

3. **[20 points]** Consider the language

$$C = \{\, w \in \Sigma^* \mid w = sbab \text{ for some } s \in \Sigma^* \,\},$$

where $\Sigma = \{a, b\}$; i.e., C consists of strings that end in bab.

(a) Give a 5-tuple description for a DFA for C. Be sure to explicit define each part of the 5-tuple for your DFA for C.

(b) Give a regular expression for C.

4. [**10 points**] Consider the language

$$D = \{\, c^i b^j c^k \mid i, j, k \geq 0,\ i = j + k \,\}.$$

Give a context-free grammar G for D. Be sure to specify G as a 4-tuple $G = (V, \Sigma, R, S)$.

5. **[10 points]** Recall the pumping lemma for regular languages:

Theorem: If L is a regular language, then there exists a pumping length p where, if $s \in L$ with $|s| \geq p$, then s can be split into three pieces $s = xyz$ such that (i) $xy^iz \in L$ for each $i \geq 0$, (ii) $|y| \geq 1$, and (iii) $|xy| \leq p$.

Let

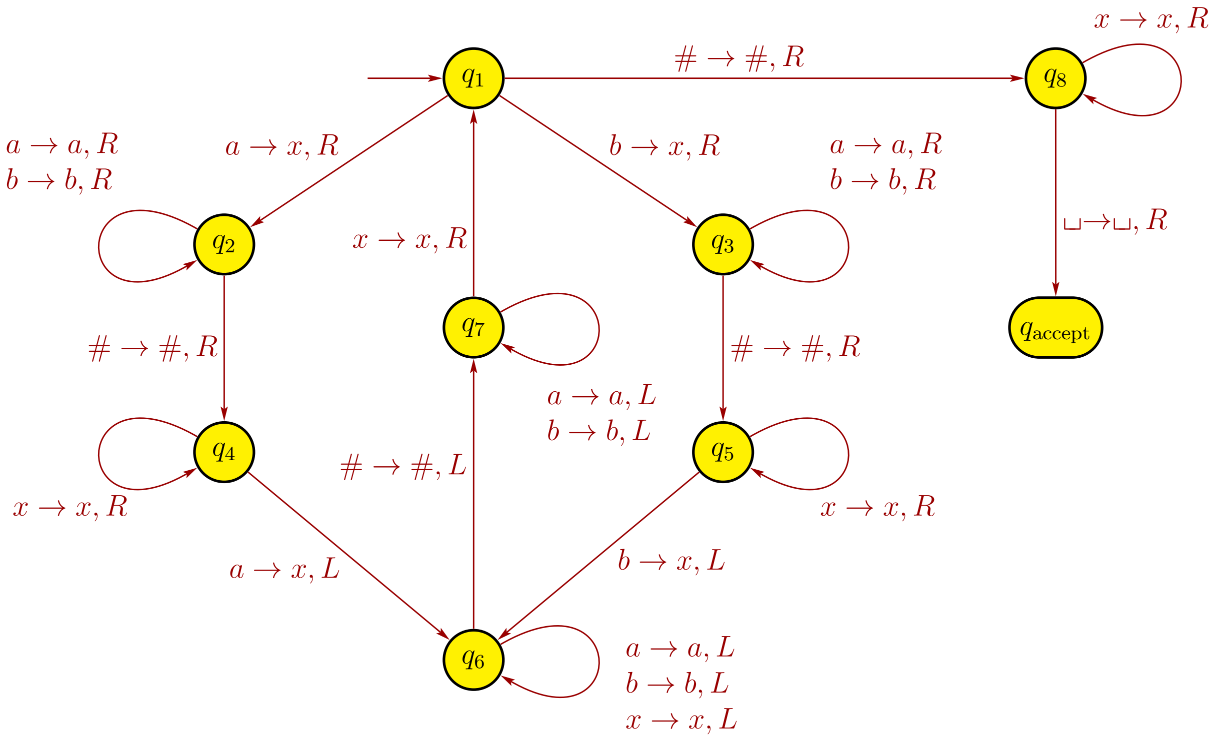
$$E = \{ w \in \Sigma^* \mid w = w^{\mathcal{R}} \text{ and } w \text{ has even length} \},$$

where $\Sigma = \{0, 1\}$, and $w^{\mathcal{R}}$ denotes the reverse of a string w. Is E a regular or nonregular language? If E is regular, give a regular expression for it. If E is not regular, prove that it is a nonregular language.

Circle one: **Regular Language** **Nonregular Language**

6. [**10 points**] Prove that the class of Turing-recognizable languages is closed under union.

7. **[10 points]** Consider the below Turing machine $M = (Q, \Sigma, \Gamma, \delta, q_1, q_{\text{accept}}, q_{\text{reject}})$ with $Q = \{q_1, \ldots, q_8, q_{\text{accept}}, q_{\text{reject}}\}$, $\Sigma = \{a, b, \#\}$, $\Gamma = \{a, b, \#, x, \sqcup\}$, and transitions below.



To simplify the figure, we don't show the reject state q_{reject} or the transitions going to the reject state. Those transitions occur implicitly whenever a state lacks an outgoing transition for a particular symbol. For example, because in state q_5 no outgoing arrow with $\#$ is present, if $\#$ occurs under the head when the machine is in state q_5, it goes to state q_{reject}. For completeness, we say that in each of these transitions to the reject state, the head writes the same symbol as is read and moves right.

Give the sequence of configurations that M enters when started on input string $bbab\#abb$.

If you need to enter a blank $\sqcup$ in your answer, you should instead use the plus symbol $+$ to represent a blank.

8. [**10 points**] The next 5 questions are of type **Multiple Answers**. There can be more than one correct answer. You will earn points for every correct answer selected, and you will lose points for every wrong answer selected.

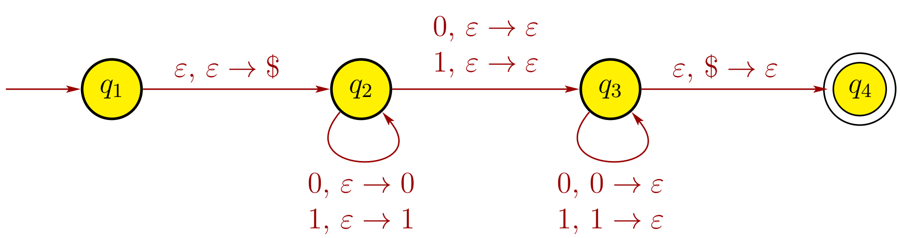
(a) Define the following classes of languages:

- D is the class of languages decided by Turing machines
- F is the class of finite languages
- G is the class of languages having context-free grammars
- N is the class of languages recognized by NFAs
- P is the class of languages recognized by PDAs
- R is the class of languages having regular expressions
- T is the class of languages recognized by Turing machines.

Which of the following relations hold?

i. F is a subset of D
ii. D is a subset of F
iii. N = R
iv. R = F
v. P is a subset of T
vi. N is a subset of G

(b) Consider a PDA $M = (Q, \Sigma, \Gamma, \delta, q_1, F)$, where $Q = \{q_1, q_2, q_3, q_4\}$, $\Sigma = \{0, 1\}$, $\Gamma = \{0, 1, \$\}$, $F = \{q_4\}$, and δ as specified in the following graph:



Which of the following CFGs (if any) generates the language $L(M)$? Below, only the rules of each CFG are specified.

i. $S \to 0S1 \mid \varepsilon$
ii. $S \to 0S0 \mid 1S1 \mid \varepsilon$
iii. $S \to 0S0 \mid 1S1 \mid 0 \mid 1$
iv. $S \to 0S0 \mid 1S1 \mid 0 \mid 1 \mid \varepsilon$
v. $S \to 0S0 \mid X \mid Y$
$X \to 1X1 \mid S$
$Y \to 0 \mid 1$
vi. None of the other answers is correct.

(c) The class of **context-free languages** is closed under

i. complementation

ii. union
iii. intersection
iv. concatenation
v. Kleene star
vi. None of the other answers is correct.

(d) The class of **finte languages** is closed under

i. union
ii. complementation
iii. intersection
iv. concatenation
v. Kleene star
vi. None of the other answers is correct.

(e) If a language A has a context-free grammar in Chomsky normal form, then A must be recognized by a

i. DFA
ii. NFA
iii. PDA
iv. Turing machine
v. k-tape Turing machine
vi. Nondeterministic Turing machine