Midterm Exam 1
CS 341: Foundations of Computer Science II — **Fall 2021, hybrid section**
Prof. Marvin K. Nakayama

Print family (or last) name: _____

Print given (or first) name: _____

I have read and understand all of the instructions below, and I will obey the University Policy on Academic Integrity.

_____
Signature and Date

- This exam has 7 pages in total, numbered 1 to 7. Make sure your exam has all the pages.

- Note the number written on the upper right-hand corner of the first page. On the sign-up sheet being passed around, print your name next to this number.

- This exam will be 1 hour and 20 minutes in length.

- This is a closed-book, closed-note exam. Electronic devices (e.g., cellphone, smart watch, calculator) are not allowed.

- For all problems, follow these instructions:

  1. Give only your answers in the spaces provided. Only what is in the answer space will be graded, and points will be deducted for any scratch work in the answer space. Use the scratch-work area or the backs of the exam sheets to work out your answers before filling in the answer space.

  2. DFA stands for deterministic finite automaton; NFA stands for nondeterministic finite automaton; PDA stands for push-down automaton; CFG stands for context-free grammar.

  3. For any state machines that you draw, you must include **all states and transitions**.

  4. For any proofs, be sure to provide a step-by-step argument, with justifications for every step. If you are asked to prove a result X, you may use in your proof of X any other result Y without proving Y. However, make it clear what the other result Y is that you are using; e.g., write something like, "By the result that $A^{**} = A^*$, we know that ...."

Problem	1	2	3	4	5	Total
Points						

1. [**20 points**] For each of the following, circle TRUE if the statement is correct. Otherwise, circle FALSE

   (a) TRUE   FALSE   —   The class of context-free languages is closed under complementation.

   (b) TRUE   FALSE   —   If a finite number of strings is added to a regular language A, then the resulting language is regular.

   (c) TRUE   FALSE   —   If a finite number of strings is added to a nonregular language A, then the resulting language is nonregular.

   (d) TRUE   FALSE   —   If A is a non-context-free language, then A is also non-regular.

   (e) TRUE   FALSE   —   If A is a regular language, then A is finite.

   (f) TRUE   FALSE   —   If A and B are regular languages, then $(\overline{A} \cup B)^*$ must be regular.

   (g) TRUE   FALSE   —   If A and B are context-free languages, then $A \cap B$ must be context-free.

   (h) TRUE   FALSE   —   If B is a context-free language and $A \subseteq B$, then A is context-free.

   (i) TRUE   FALSE   —   The language $\{\, a^n b^n \mid n \leq 30 \}$ is non-regular.

   (j) TRUE   FALSE   —   If a language A has a regular expression, then A has a CFG in Chomsky normal form.
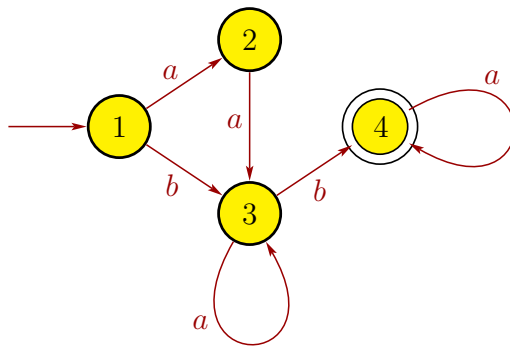
2. [**20 points**]   Give short answers to each of the following parts. Each answer should be at most a few sentences. Be sure to define any notation that you use.

   (a) Let $\Sigma = \{0, 1\}$, and let A be the set of strings over Σ having an odd number of 1's. Give a regular expression for A.
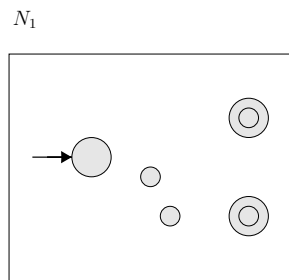
   Answer: _____

   (b) Give a regular expression for the language recognized by the NFA below.

   Answer: _____

(c) Suppose that language A_1 is recognized by NFA N_1 below. Note that the transitions are not drawn in N_1. Draw a picture of an NFA for A_1^*.

N_1

(d) Suppose that A_1 is a language defined by a CFG $G_1 = (V_1, \Sigma, R_1, S_1)$, and A_2 is a language defined by a CFG $G_2 = (V_2, \Sigma, R_2, S_2)$, where the alphabet Σ is the same for both languages and $V_1 \cap V_2 = \emptyset$. Let $A_3 = A_1 \circ A_2$. Give a CFG G_3 for A_3 in terms of G_1 and G_2. You do not have to prove the correctness of your CFG G_3, but do not give just an example.

3. **[15 points]** Let N be the following NFA with $\Sigma = \{a, b\}$, and let $C = L(N)$.

a
a
1   2
ε a b
3   4
a
b
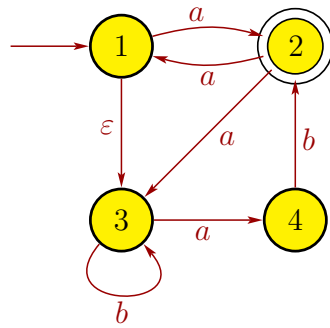
Give a DFA for C. You only need to draw the graph; do not give the 5-tuple.

Scratch-work area

5

4. **[30 points]** Consider the alphabet $\Sigma = \{a, b, c\}$ and the language

$$L = \{\, c^i a^j b^k \mid i, j, k \geq 0 \text{ and } i + j = k \,\}.$$

(a) Give a context-free grammar G for L. Be sure to specify G as a 4-tuple $G = (V, \Sigma, R, S)$.

(b) Give a PDA for L. You only need to draw the graph.

Scratch-work area

5. **[15 points]** Recall the pumping lemma for regular languages:

Theorem: If L is a regular language, then there exists a pumping length p where, if $s \in L$ with $|s| \geq p$, then s can be split into three pieces $s = xyz$ such that (i) $xy^i z \in L$ for each $i \geq 0$, (ii) $|y| \geq 1$, and (iii) $|xy| \leq p$.

Let $A = \{\, www \mid w \in \Sigma^* \,\}$ for $\Sigma = \{a, b\}$. Is A a regular or nonregular language? If A is regular, give a regular expression for A. If A is not regular, prove that it is a nonregular language.

Circle one: **Regular Language**   **Nonregular Language**