Midterm Exam 1
CS 341-006: Foundations of Computer Science II — **Spring 2021, hybrid section**
Prof. Marvin K. Nakayama

Print family (or last) name: _____

Print given (or first) name: _____

I have read and understand all of the instructions below, and I will obey the University Policy on Academic Integrity.

_____

Signature and Date

- This exam has 7 pages in total, numbered 1 to 7. Make sure your exam has all the pages.

- Note the number written on the upper right-hand corner of the first page. On the sign-up sheet being passed around, print your name next to this number.

- This exam will be 1 hour and 20 minutes in length.

- This is a closed-book, closed-note exam. Electronic devices (e.g., cellphone, smart watch, calculator) are not allowed.

- For all problems, follow these instructions:

  1. Give only your answers in the spaces provided. I will only grade what you put in the answer space, and I will take off points for any scratch work in the answer space. Use the scratch-work area or the backs of the exam sheets to work out your answers before filling in the answer space.

  2. DFA stands for deterministic finite automaton; NFA stands for nondeterministic finite automaton; PDA stands for push-down automaton; CFG stands for context-free grammar.

  3. For any state machines that you draw, you must include **all states and transitions**.

  4. For any proofs, be sure to provide a step-by-step argument, with justifications for every step. If you are asked to prove a result X, you may use in your proof of X any other result Y without proving Y. However, make it clear what the other result Y is that you are using; e.g., write something like, "By the result that $A^{**} = A^*$, we know that ...."

| Problem	1	2	3	4	5	Total
Points						

1. [**10 points**]   For each of the following, circle TRUE if the statement is correct. Otherwise, circle FALSE

(a)   TRUE   FALSE   —   If a language A is context-free, then A has an NFA.

(b)   TRUE   FALSE   —   If A is a regular language and B is a nonregular language, then $A \cup B$ must be nonregular.

(c)   TRUE   FALSE   —   If A and B are nonregular languages, then $A \cap B$ must be nonregular.

(d)   TRUE   FALSE   —   If A is a regular language, then A must be finite.

(e)   TRUE   FALSE   —   If A is a nonregular language, then $\overline{A}$ must be nonregular.

(f)   TRUE   FALSE   —   If A and B are context-free languages, then $A \cap B$ must also be context-free.

(g)   TRUE   FALSE   —   A context-free grammar for $A = \{\, 0^n 1^n 0^n \mid n \geq 0 \,\}$ is $G = (V, \Sigma, R, S)$, with $V = \{S\}$, $\Sigma = \{0, 1\}$, start variable S, and rules $R = \{\, S \rightarrow 0S1S0S \mid \varepsilon \,\}$.

(h)   TRUE   FALSE   —   If a language A has a regular expression, then A must have a PDA.

(i)   TRUE   FALSE   —   If a language A is nonregular, then A must have a context-free grammar in Chomsky normal form.

(j)   TRUE   FALSE   —   The class of languages recognized by NFAs is closed under concatenation.

2. **[40 points]** Give short answers to each of the following parts. Each answer should be at most a few sentences. Be sure to define any notation that you use.

  (a) Give a regular expression for the language recognized by the NFA below.
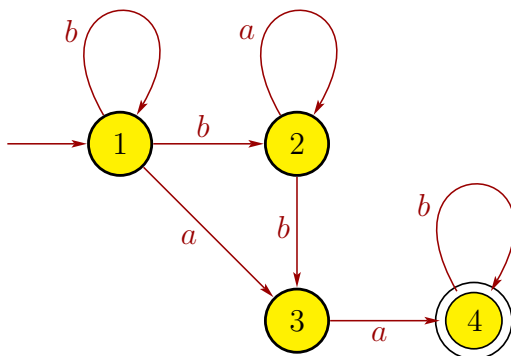
  (b) Suppose a language A_1 is generated by a context-free grammar $G_1 = (V_1, \Sigma, R_1, S_1)$, and a language A_2 is generated by a context-free $G_2 = (V_2, \Sigma, R_2, S_2)$, where for $i = 1, 2$, we have that V_i is the set of variables in G_i, R_i is the set of rules in G_i, and S_1 is the start variable of G_i. The two grammars share the same set Σ of terminals, and $V_1 \cap V_2 = \emptyset$. Give a context-free grammar G_3 for $A_1 \cup A_2$ in terms of G_1 and G_2. You do not have to prove the correctness of your CFG G_3, but do not just give an example.

(c) Let $M_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$ be a DFA with language A_1, and $M_2 = (Q_2, \Sigma, \delta_2, q_2, F_2)$ be a DFA with language A_2. Consider the language $A = A_1 \cap A_2$. Give a DFA M_3 for A in terms of M_1 and M_2. Your DFA M_3 must be completely general. Do not prove the correctness of your DFA M_3, but do not just give an example.

(d) Suppose that we are in the process of converting a CFG G with $\Sigma = \{0, 1\}$ into Chomsky normal form. We have already applied some steps in the process, and we currently have the following CFG:

$$
\begin{aligned}
S_0 &\rightarrow S \\
S &\rightarrow 0A1SA \mid \varepsilon \\
A &\rightarrow 0S0 \mid A0S10S1 \mid \varepsilon
\end{aligned}
$$

In the next step, we want to remove the ε-rule $S \rightarrow \varepsilon$. Give the CFG after carrying out just this one step.

3. [**20 points**]   Consider the language

$$C = \{\, w \in \Sigma^* \mid w = sbba \text{ for some } s \in \Sigma^* \,\},$$

where $\Sigma = \{a, b\}$; i.e., C consists of strings that end in *bba*.

(a) Give a 5-tuple description for a DFA for C. Be sure to explicit define each part of the 5-tuple for your DFA for C.

(b) Give a regular expression for C.

4. [**15 points**]   Consider the language

$$D = \{\, a^i b^j \mid i \leq j \,\}.$$

Give a context-free grammar G for D. Be sure to specify G as a 4-tuple $G = (V, \Sigma, R, S)$.

5. [**15 points**] Recall the pumping lemma for regular languages:

   Theorem: If L is a regular language, then there exists a pumping length p where, if $s \in L$ with $|s| \geq p$, then s can be split into three pieces $s = xyz$ such that (i) $xy^i z \in L$ for each $i \geq 0$, (ii) $|y| \geq 1$, and (iii) $|xy| \leq p$.

   Let $E = \{\, w \in \Sigma^* \mid w = w^{\mathcal{R}} \,\}$, where $\Sigma = \{0, 1\}$, and $w^{\mathcal{R}}$ denotes the reverse of a string w. Is E a regular or nonregular language? If E is regular, give a regular expression for it. If E is not regular, prove that it is a nonregular language.

   Circle one: **Regular Language**   **Nonregular Language**