Midterm Exam 2
CS 341-006: Foundations of Computer Science II — **Spring 2021, Hybrid section**
Prof. Marvin K. Nakayama

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I have read and understand all of the instructions below, and I will obey the University Policy on
Academic Integrity.

Signature and Date: ⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯

- This exam has 8 pages in total, numbered 1 to 8. Make sure your exam has all the pages.

- This exam will be 1 hour and 20 minutes in length.

- This is a closed-book, closed-note exam. Electronic devices (e.g., cellphone, smart watch,
  calculator) are not allowed.

- For all problems, follow these instructions:

  1. Give only your answers in the spaces provided. I will only grade what you put in the
     answer space, and I will take off points for any scratch work in the answer space. Use
     the scratch-work area or the backs of the sheets to work out your answers before filling
     in the answer space.
  2. DFA stands for deterministic finite automaton; NFA stands for nondeterministic finite
     automaton; CFG stands for context-free grammar; PDA stands for pushdown automaton;
     TM stands for Turing machine.
  3. For any state machines that you draw, you must include **all states and transitions**.
  4. For any proofs, be sure to provide a step-by-step argument, with justifications for every
     step. Unless you are specifically asked to prove a theorem from the book or notes, you
     may assume that the theorems in the textbook and notes hold; i.e., you do not have
     to reprove the theorems in the textbook and notes. When using a theorem from the
     textbook or notes, make sure you provide enough detail so that it is clear which result
     you are using; e.g., say something like, "By the theorem that states $S^{**} = S^*$, it follows
     that ..."

Problem	1	2	3	4	5	6	Total
Points							

1. **[20 points]** For each of the following, circle TRUE if the statement is correct. Otherwise, circle FALSE

(a) TRUE   FALSE — Every language is Turing-recognizable.

(b) TRUE   FALSE — The universal Turing machine decides

$$A_{\text{TM}} = \{ \langle M, w \rangle \mid M \text{ is a TM that accepts string } w \}.$$

(c) TRUE   FALSE — If languages A and B are unequal, then $\overline{A} \cap B \neq \emptyset$ and $A \cap \overline{B} \neq \emptyset$.

(d) TRUE   FALSE — If a set A is infinite, then A is uncountable.

(e) TRUE   FALSE — Every finite language is Turing-recognizable.

(f) TRUE   FALSE — Running a Turing machine $M = (Q, \Sigma, \Gamma, \delta, q_1, q_{\text{accept}}, q_{\text{reject}})$ on a string $w \in \Sigma^*$ with $w \notin L(M)$ will result in M ending in q_{reject}.

(g) TRUE   FALSE — The power set of a countable set is countable.

(h) TRUE   FALSE — Every enumerator has an equivalent single-tape Turing machine.

(i) TRUE   FALSE — The set of all Turing machines is finite.

(j) TRUE   FALSE — If a set B is uncountable and $A \subseteq B$, then A must be uncountable.

2. **[20 points]**   Give a short answer (at most three sentences) for each part below. For parts (a), (b) and (c), let $D = \{a, b, c\}$ and $R = \{2, 4, 6, 8\}$, and define the function $f : D \to R$ such that

$$\begin{aligned} f(a) &= 6, \\ f(b) &= 8, \\ f(c) &= 4. \end{aligned}$$

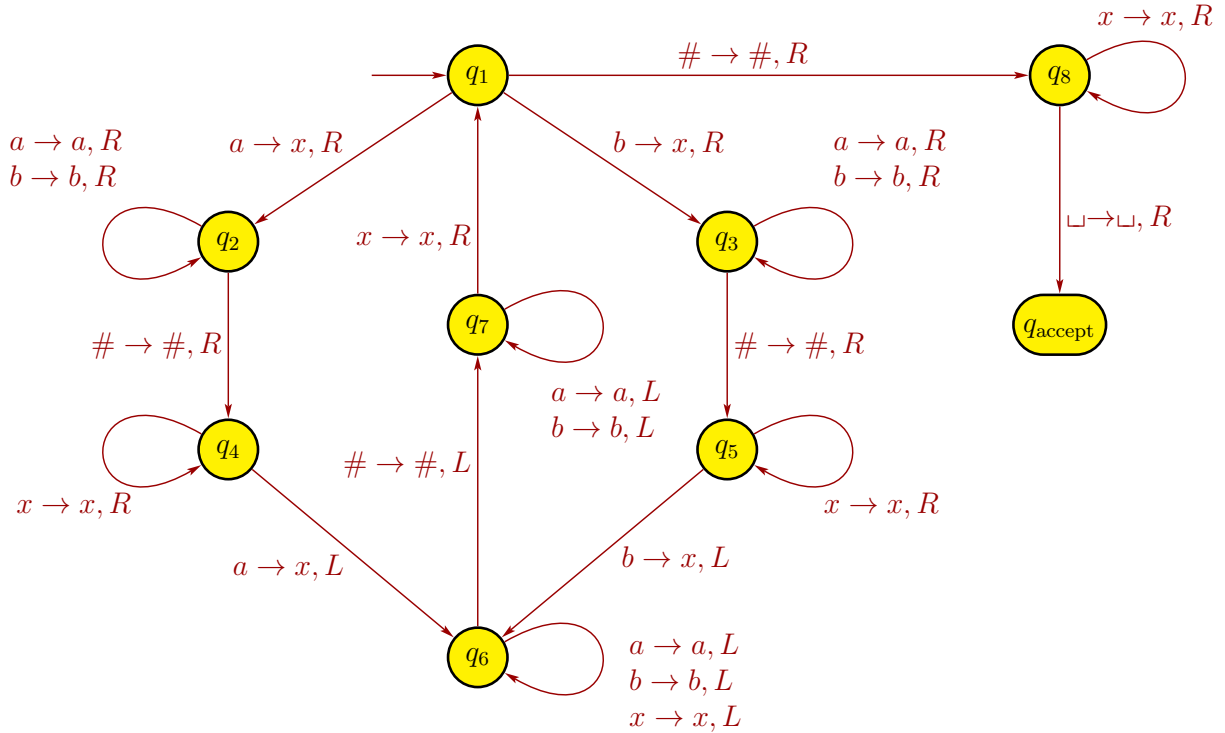Explain your answers.

(a) Is f one-to-one?

(b) Is f onto?

(c) Is f a correspondence?

(d) What is the difference between a Turing-recognizable language and a Turing-decidable language?

(e) What does the Church-Turing Thesis say?

3. **[15 points]** Consider the below Turing machine $M = (Q, \Sigma, \Gamma, \delta, q_1, q_{\text{accept}}, q_{\text{reject}})$ with $Q = \{q_1, \ldots, q_8, q_{\text{accept}}, q_{\text{reject}}\}$, $\Sigma = \{a, b, \#\}$, $\Gamma = \{a, b, \#, x, \sqcup\}$, and transitions below.

To simplify the figure, we don't show the reject state q_{reject} or the transitions going to the reject state. Those transitions occur implicitly whenever a state lacks an outgoing transition for a particular symbol. For example, because in state q_5 no outgoing arrow with $\#$ is present, if $\#$ occurs under the head when the machine is in state q_5, it goes to state q_{reject}. For completeness, we say that in each of these transitions to the reject state, the head writes the same symbol as is read and moves right.

Give the sequence of configurations that M enters when started on input string *abba#baba*.

If you need to enter a blank $\sqcup$ in your answer, you should instead use the plus symbol $+$ to represent a blank.

Each of the following problems requires you to prove a result. If you are asked to prove a result A and your proof relies on another result B, then you do not need to prove B if B is a result that we either went over in class or was in the homework. In this case, you need to make clear what result B you are citing in your proof of A (e.g., say something like, "By the result that $S^{**} = S^*$ for any set S of strings, we can show that ..."). If the result B has not been covered in class or in the homework, then you will also have to prove B.

4. [**15 points**]   Prove that the set of all languages over an alphabet Σ is uncountable.

5. [**15 points**] Prove that if a language A is Turing-recognizable, then there is an enumerator that enumerates A.

6. [**15 points**]   Consider the problem of determining **whether an NFA with alphabet** $\Sigma = \{0, 1\}$ **accepts at least one string that has** 101 **as a substring.** Express this problem as a language and show that it is decidable.