Midterm Exam 2
CS 341-008: Foundations of Computer Science II — **Spring 2021, Hybrid section**
Prof. Marvin K. Nakayama

Print family (or last) name: ____________________________________

Print given (or first) name: ____________________________________

I have read and understand all of the instructions below, and I will obey the University Policy on Academic Integrity.

Signature and Date: ____________________________________

- This exam has 8 pages in total, numbered 1 to 8. Make sure your exam has all the pages.
- This exam will be 1 hour and 20 minutes in length.
- This is a closed-book, closed-note exam. Electronic devices (e.g., cellphone, smart watch, calculator) are not allowed.
- For all problems, follow these instructions:
  1. Give only your answers in the spaces provided. I will only grade what you put in the answer space, and I will take off points for any scratch work in the answer space. Use the scratch-work area or the backs of the sheets to work out your answers before filling in the answer space.
  2. DFA stands for deterministic finite automaton; NFA stands for nondeterministic finite automaton; CFG stands for context-free grammar; PDA stands for pushdown automaton; TM stands for Turing machine.
  3. For any state machines that you draw, you must include **all states and transitions**.
  4. For any proofs, be sure to provide a step-by-step argument, with justifications for every step. Unless you are specifically asked to prove a theorem from the book or notes, you may assume that the theorems in the textbook and notes hold; i.e., you do not have to reprove the theorems in the textbook and notes. When using a theorem from the textbook or notes, make sure you provide enough detail so that it is clear which result you are using; e.g., say something like, "By the theorem that states $S^{**} = S^*$, it follows that ..."

Problem	1	2	3	4	5	6	Total
Points							

1. **[20 points]** For each of the following, circle TRUE if the statement is correct. Otherwise, circle FALSE

(a) TRUE FALSE — Each subset of a countable set is countable

(b) TRUE FALSE — The universal Turing machine decides

$$A_{\text{TM}} = \{ \langle M, w \rangle \mid M \text{ is a TM that accepts string } w \}.$$

(c) TRUE FALSE — If a set A is infinite, then A is uncountable.

(d) TRUE FALSE — Every nondeterministic Turing machine has an equivalent multi-tape Turing machine.

(e) TRUE FALSE — Every regular language is not Turing-recognizable.

(f) TRUE FALSE — Each subset of an uncountable set is uncountable

(g) TRUE FALSE — If languages A and B satisfy $\overline{A} \cap B = \emptyset$ or $A \cap \overline{B} = \emptyset$, then A and B are equal.

(h) TRUE FALSE — The set of all Turing machines is uncountable.

(i) TRUE FALSE — If a language A is recognized by a Turing machine that loops on some string $w \notin A$, then A is Turing-recognizable but not decidable.

(j) TRUE FALSE — For any Turing machine $M = (Q, \Sigma, \Gamma, \delta, q_1, q_{\text{accept}}, q_{\text{reject}})$ and string $w \in \Sigma^*$, M will accept or reject on w.

2. **[20 points]** Give a short answer (at most three sentences) for each part below. For parts (a), (b) and (c), let $D = \{1, 2, 3\}$ and $R = \{a, b\}$, and define the function $f : D \to R$ such that

$$\begin{aligned} f(1) &= b, \\ f(2) &= a, \\ f(3) &= b. \end{aligned}$$

Explain your answers.

(a) Is f one-to-one?

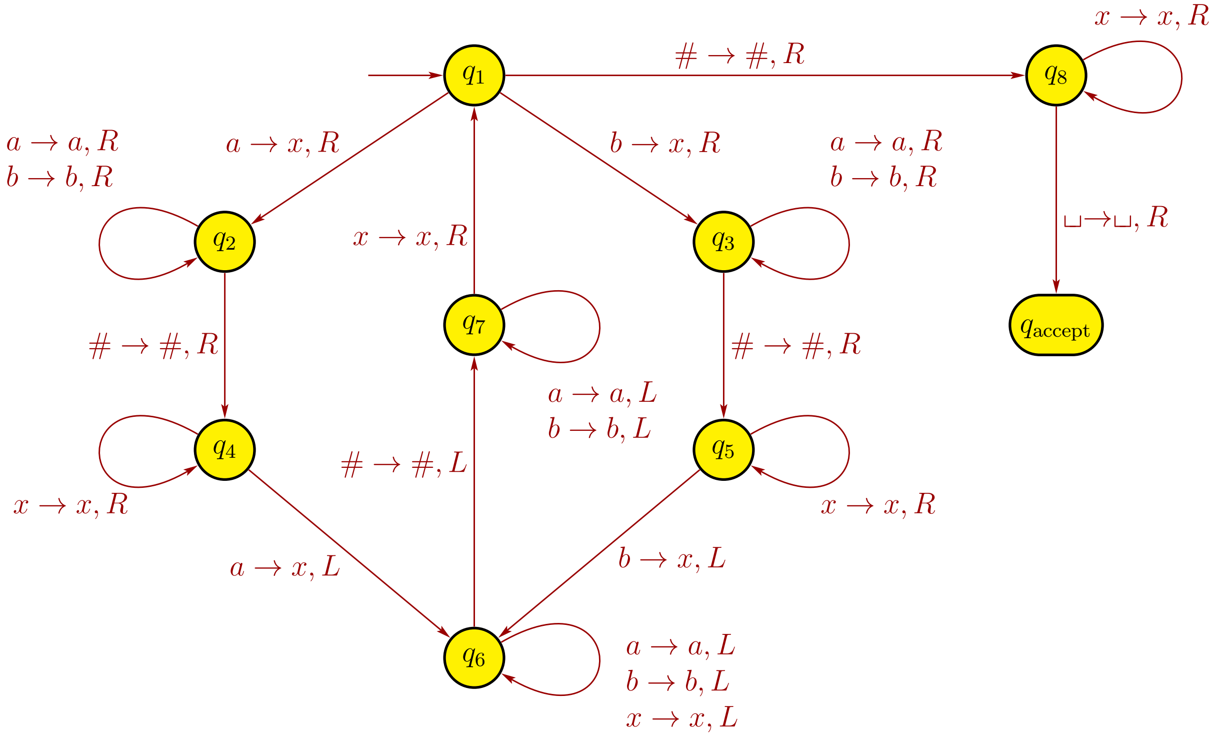
(b) Is f onto?

(c) Is f a correspondence?

(d) What is the difference between a Turing-recognizable language and a Turing-decidable language?

(e) What does the Church-Turing Thesis say?

3. **[15 points]** Consider the below Turing machine $M = (Q, \Sigma, \Gamma, \delta, q_1, q_{\text{accept}}, q_{\text{reject}})$ with $Q = \{q_1, \ldots, q_8, q_{\text{accept}}, q_{\text{reject}}\}$, $\Sigma = \{a, b, \#\}$, $\Gamma = \{a, b, \#, x, \sqcup\}$, and transitions below.



To simplify the figure, we don't show the reject state q_{reject} or the transitions going to the reject state. Those transitions occur implicitly whenever a state lacks an outgoing transition for a particular symbol. For example, because in state q_5 no outgoing arrow with # is present, if # occurs under the head when the machine is in state q_5, it goes to state q_{reject}. For completeness, we say that in each of these transitions to the reject state, the head writes the same symbol as is read and moves right.

Give the sequence of configurations that M enters when started on input string $baab\#aaba$.

If you need to enter a blank $\sqcup$ in your answer, you should instead use the plus symbol + to represent a blank.

Each of the following problems requires you to prove a result. If you are asked to prove a result A and your proof relies on another result B, then you do not need to prove B if B is a result that we either went over in class or was in the homework. In this case, you need to make clear what result B you are citing in your proof of A (e.g., say something like, "By the result that $S^{**} = S^*$ for any set S of strings, we can show that ..."). If the result B has not been covered in class or in the homework, then you will also have to prove B.

4. **[15 points]** Prove that the set of all infinite binary sequences is uncountable. Explain your answer.

5. **[15 points]** Consider the problem of determining whether a context-free grammar generates ε. Express this problem as a language and show that it is decidable.

6. **[15 points]** Consider the emptiness problem for Turing machines:

$$E_{\mathrm{TM}} = \{ \langle M \rangle \mid M \text{ is a Turing machine with } L(M) = \emptyset \}.$$

Show that E_{TM} is co-Turing-recognizable.