

Midterm Exam 1

CS 341: Foundations of Computer Science II — **Spring 2022, hybrid section**

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Print family (or last) name: _____

Print given (or first) name: _____

I have read and understand all of the instructions below, and I will obey the University Policy on Academic Integrity.

Signature and Date

- This exam has 7 pages in total, numbered 1 to 7. Make sure your exam has all the pages.
- Note the number written on the upper right-hand corner of the first page. On the sign-up sheet being passed around, print your name next to this number.
- This exam will be 1 hour and 20 minutes in length.
- This is a closed-book, closed-note exam. Electronic devices (e.g., cellphone, smart watch, calculator) are not allowed.
- For all problems, follow these instructions:
 1. Give only your answers in the spaces provided. Only what is in the answer space will be graded, and points will be deducted for any scratch work in the answer space. Use the scratch-work area or the backs of the exam sheets to work out your answers before filling in the answer space.
 2. DFA stands for deterministic finite automaton; NFA stands for nondeterministic finite automaton; PDA stands for push-down automaton; CFG stands for context-free grammar.
 3. For any state machines that you draw, you must include **all states and transitions**.
 4. For any proofs, be sure to provide a step-by-step argument, with justifications for every step. If you are asked to prove a result X, you may use in your proof of X any other result Y without proving Y. However, make it clear what the other result Y is that you are using; e.g., write something like, “By the result that $A^{**} = A^*$, we know that”

Problem	1	2	3	4	5	Total
Points						

1. [20 points] For each of the following, circle TRUE if the statement is correct. Otherwise, circle FALSE

- (a) TRUE FALSE — If a finite number of strings is added to a regular language A , then the resulting language must be regular.
- (b) TRUE FALSE — If a finite number of strings is added to a nonregular language A , then the resulting language must be nonregular.
- (c) TRUE FALSE — If A is a non-context-free language, then A is also non-regular.
- (d) TRUE FALSE — If $A \subseteq B$, then $A \cap \overline{B} = \emptyset$.
- (e) TRUE FALSE — If B is a context-free language and $A \subseteq B$, then A is context-free.
- (f) TRUE FALSE — The language $\{a^n b^n \mid 5 \leq n \leq 20\}$ is non-regular.
- (g) TRUE FALSE — If a language A has a regular expression, then A must have a CFG in Chomsky normal form.
- (h) TRUE FALSE — The class of context-free languages is closed under union.
- (i) TRUE FALSE — If A and B are regular languages, then $(\overline{A} \cup B)^*$ must be regular.
- (j) TRUE FALSE — The class of context-free languages is closed under intersection.

2. **[20 points]** Give short answers to each of the following parts. Each answer should be at most a few sentences. Be sure to define any notation that you use.

(a) For the alphabet $\Sigma = \{a, b\}$, let $A = \{w \in \Sigma^* \mid \text{number of } b\text{'s in } w \text{ is odd}\}$. Give a regular expression for A .

(b) Suppose a language A_1 is generated by a context-free grammar $G_1 = (V_1, \Sigma, R_1, S_1)$. Give a context-free grammar G_2 for A_1^* in terms of G_1 . You do not have to prove the correctness of your CFG G_2 , but do not just give an example.

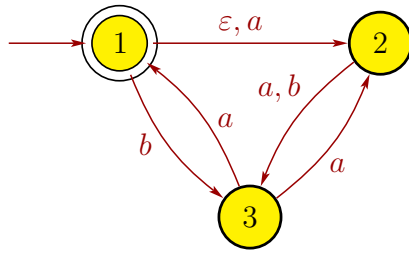
- (c) Let $M_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$ be a DFA with language A_1 , and $M_2 = (Q_2, \Sigma, \delta_2, q_2, F_2)$ be a DFA with language A_2 . Consider the language $A = A_1 \cap A_2$. Give a DFA M_3 for A in terms of M_1 and M_2 . Your DFA M_3 must be completely general. Do not prove the correctness of your DFA M_3 , but do not just give an example.

- (d) Suppose that we are in the process of converting a CFG G with $\Sigma = \{0, 1\}$ into Chomsky normal form. We have already applied some steps in the process, and we currently have the following CFG:

$$\begin{aligned} S_0 &\rightarrow S \\ S &\rightarrow 0SA0SA \mid 0A0S \mid \varepsilon \\ A &\rightarrow 10A01 \mid \varepsilon \end{aligned}$$

In the next step, we want to remove the ε -rule $A \rightarrow \varepsilon$. Give the CFG after carrying out **just this one step**.

3. [20 points] For $\Sigma = \{a, b\}$, let C be the language recognized by the following NFA:



(a) [5 points] List the first 5 strings in C in string order.

(b) [15 points] Give a DFA for C . You only need to draw the graph.

Scratch-work area

4. [25 points] Consider the language

$$D = \{ c^i a^j b^k \mid i, j, k \geq 0, \text{ and } j = i + k \}.$$

(a) Give a context-free grammar G for D . Be sure to specify G as a 4-tuple $G = (V, \Sigma, R, S)$.

(b) Give a PDA for D . You only need to draw the graph.

Scratch-work area

5. [15 points] Recall the pumping lemma for regular languages:

Theorem: If L is a regular language, then there exists a pumping length p where, if $s \in L$ with $|s| \geq p$, then s can be split into three pieces $s = xyz$ such that (i) $xy^iz \in L$ for each $i \geq 0$, (ii) $|y| \geq 1$, and (iii) $|xy| \leq p$.

Let $D = \{c^i a^j b^k \mid i, j, k \geq 0, \text{ and } j = i + k\}$. Is D a regular or nonregular language? If D is regular, give a regular expression for D . If D is not regular, prove that it is a nonregular language.

Circle one:

Regular Language

Nonregular Language