

Midterm Exam 2

CS 341-006: Foundations of Computer Science II — **Spring 2022, Hybrid section**

Prof. Marvin K. Nakayama

Print family (or last) name: \_\_\_\_\_

Print given (or first) name: \_\_\_\_\_

I have read and understand all of the instructions below, and I will obey the University Policy on Academic Integrity.

Signature and Date: \_\_\_\_\_

- This exam has 9 pages in total, numbered 1 to 9. Make sure your exam has all the pages.
- This exam will be 1 hour and 20 minutes in length.
- This is a closed-book, closed-note exam. Electronic devices (e.g., cellphone, smart watch, calculator, headphones) are not allowed.
- For all problems, follow these instructions:
  1. Give only your answers in the spaces provided. Only what you put in the answer space will be graded, and points will be deducted for any scratch work in the answer space. Use the scratch-work area or the backs of the sheets to work out your answers before filling in the answer space.
  2. DFA stands for deterministic finite automaton; NFA stands for nondeterministic finite automaton; CFG stands for context-free grammar; PDA stands for pushdown automaton; TM stands for Turing machine.
  3. For any state machines that you draw, you must include **all states and transitions**.
  4. For any proofs, be sure to provide a step-by-step argument, with justifications for every step. Unless you are specifically asked to prove a theorem from the book or notes, you may assume that the theorems in the textbook and notes hold; i.e., you do not have to reprove the theorems in the textbook and notes. When using a theorem from the textbook or notes, make sure you provide enough detail so that it is clear which result you are using; e.g., say something like, “By the theorem that states  $S^{**} = S^*$ , it follows that ...”

Problem	1	2	3	4	5	6	7	Total
Points								

1. [20 points] For each of the following, circle TRUE if the statement is correct. Otherwise, circle FALSE

(a) TRUE FALSE — A deterministic Turing machine can recognize an undecidable language.

(b) TRUE FALSE — There are some languages recognized by a 5-tape, nondeterministic Turing machine that cannot be recognized by a 1-tape, deterministic Turing machine.

(c) TRUE FALSE — For any Turing machine  $M = (Q, \Sigma, \Gamma, \delta, q_1, q_{\text{accept}}, q_{\text{reject}})$  and string  $w \in \Sigma^*$ , the TM  $M$  will either accept or reject  $w$ .

(d) TRUE FALSE — If a language is context-free, then it must be Turing-decidable.

(e) TRUE FALSE — The set of all Turing machines is countable.

(f) TRUE FALSE — The problem of determining if a context-free grammar generates the empty language is undecidable.

(g) TRUE FALSE — Every language is Turing-recognizable.

(h) TRUE FALSE — For sets  $A$  and  $B$ ,  $A \subseteq B$  if and only if  $A \cap \overline{B} = \emptyset$ .

(i) TRUE FALSE — The language

$$EQ_{\text{DFA}} = \{ \langle C, D \rangle \mid C \text{ and } D \text{ are DFAs with } L(C) = L(D) \}$$

is Turing-decidable.

(j) TRUE FALSE — The universal Turing machine decides

$$A_{\text{TM}} = \{ \langle M, w \rangle \mid M \text{ is a TM that accepts string } w \}.$$

2. [20 points] Give a short answer (at most three sentences) for each part below. For parts (a), (b) and (c), let  $D = \{a, b, c, d\}$  and  $R = \{1, 2, 3, 4\}$ , and define the function  $f : D \rightarrow R$  such that

$$f(a) = 1,$$

$$f(b) = 4,$$

$$f(c) = 2,$$

$$f(d) = 3.$$

Explain your answers.

(a) Is  $f$  one-to-one?

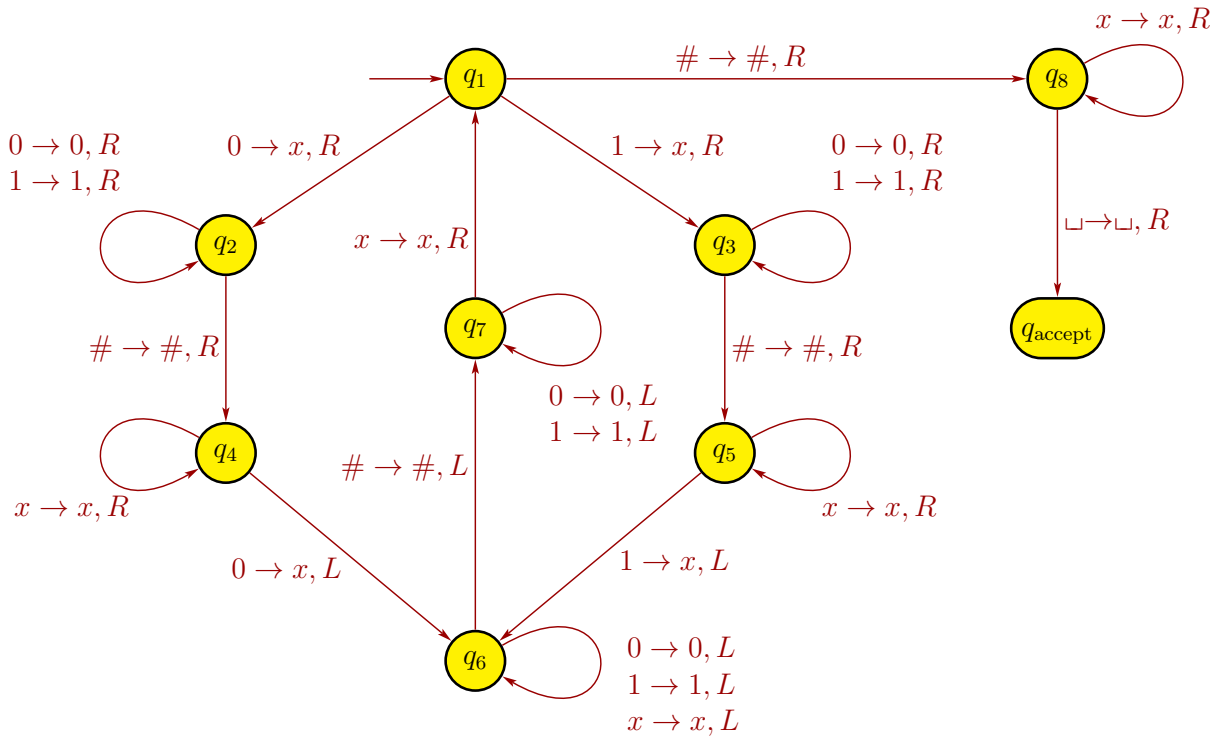
(b) Is  $f$  onto?

(c) Is  $f$  a correspondence?

(d) What is the difference between a Turing-recognizable language and a Turing-decidable language?

(e) What does the Church-Turing Thesis say?

3. [10 points] Consider the below Turing machine  $M = (Q, \Sigma, \Gamma, \delta, q_1, q_{\text{accept}}, q_{\text{reject}})$  with  $Q = \{q_1, \dots, q_8, q_{\text{accept}}, q_{\text{reject}}\}$ ,  $\Sigma = \{0, 1, \#\}$ ,  $\Gamma = \{0, 1, \#, x, \sqcup\}$ , and transitions below. To simplify the figure, we don't show the reject state  $q_{\text{reject}}$  or the transitions going to the reject state. Those transitions occur implicitly whenever a state lacks an outgoing transition for a particular symbol. For example, because in state  $q_5$  no outgoing arrow with a  $\#$  is present, if a  $\#$  occurs under the head when the machine is in state  $q_5$ , it goes to state  $q_{\text{reject}}$ . For completeness, we say that in each of these transitions to the reject state, the head writes the same symbol as is read and moves right.



Give the sequence of configurations that  $M$  enters when started on input string 0001#10.

Each of the following problems requires you to prove a result. If you are asked to prove a result  $A$  and your proof relies on another result  $B$ , then you do not need to prove  $B$  if  $B$  is a result that we either went over in class or was in the homework. In this case, you need to make clear what result  $B$  you are citing in your proof of  $A$  (e.g., say something like, “By the result that  $S^{**} = S^*$  for any set  $S$  of strings, we can show that ...”).

4. **[10 points]** Let  $A = \{x \in \mathfrak{R} \mid 6 \leq x < 7\}$ , where  $\mathfrak{R}$  is the set of real numbers. Prove that  $A$  is uncountable.

5. [10 points] Show that the class of Turing-recognizable languages is closed under union. Explain your answer.

6. [15 points] Consider the decision problem of determining **for two DFAs if every string accepted by the first DFA is also accepted by the second DFA**. Express this problem as a language and show that it is decidable. You may assume that the two DFAs share the same alphabet  $\Sigma$ .



7. [15 points] Recall the language for the emptiness problem for Turing machines:

$$E_{\text{TM}} = \{ \langle M \rangle \mid M \text{ is a Turing machine with } L(M) = \emptyset \}.$$

Show that  $\overline{E_{\text{TM}}}$  is Turing-recognizable.