Midterm Exam
CS 341-452: Foundations of Computer Science II — **Spring 2022, eLearning (online) section**
Prof. Marvin K. Nakayama

Print family (or last) name: ____________________________________

Print given (or first) name: ____________________________________

I have read and understand all of the instructions below, and I will obey the University Policy on Academic Integrity.

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Signature and Date

- This exam has 11 pages in total, numbered 1 to 11. Make sure your exam has all the pages.
- Unless other arrangements have been made with the professor, the exam is to last 2.5 hours. and is to be given on Saturday, 3/12/2022.
- This is a closed-book, closed-note exam. Electronic devices (e.g., cellphone, smart watch, calculator) are not allowed.
- For all problems, follow these instructions:
  1. Give only your answers in the spaces provided. I will only grade what you put in the answer space, and I will take off points for any scratch work in the answer space. Use the scratch-work area or the backs of the sheets to work out your answers before filling in the answer space.
  2. DFA stands for deterministic finite automaton; NFA stands for nondeterministic finite automaton; CFG stands for context-free grammar; PDA stands for pushdown automaton; TM stands for Turing machine.
  3. For any state machines that you draw, you must include **all states and transitions**, unless otherwise specified.
  4. For any proofs, be sure to provide a step-by-step argument, with justifications for every step. If you are asked to prove a result X, in your proof of X, you may use any other result Y without proving Y. However, make it clear what the other result Y is that you are using; e.g., write something like, "By the result that $A^{**} = A^*$, we know that ...."

Problem	1	2	3	4	5	6	7	8	Total
Points									

1. **[10 points]** For each of the following, circle TRUE if the statement is correct. Otherwise, circle FALSE

(a) TRUE FALSE — If A is a regular language, then A must be finite.

(b) TRUE FALSE — A language that is decided by a nondeterministic Turing machine must also be recognized by a deterministic Turing machine.

(c) TRUE FALSE — If a language A is recognized by a $M = (Q, \Sigma, \delta, q_1, F)$, then A^* is recognized by the DFA $M' = (Q, \Sigma, \delta, q_1, Q - F)$.

(d) TRUE FALSE — If a language A is recognized by a 5-tape Turing machine, then $A \cup \overline{A}$ must be regular.

(e) TRUE FALSE — A context-free grammar for $A = \{ 1^n 0^n \mid n \geq 0\}$ is $G = (V, \Sigma, R, S)$, with $V = \{S\}$, $\Sigma = \{0, 1\}$, start variable S, and rules $R = \{ S \to 1S0S \mid \varepsilon \}$.

(f) TRUE FALSE — If a language A is finite, then $\overline{A}$ must have a context-free grammar in Chomsky normal form.

(g) TRUE FALSE — If a language A is recognized by an NFA, then A^* must be infinite.

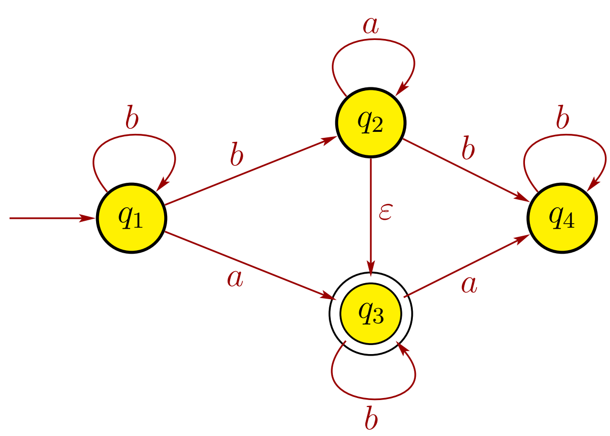
(h) TRUE FALSE — Every language recognized by a Turing machine has a CFG.

(i) TRUE FALSE — If A and B are regular languages, then $\overline{A} \circ B$ must be regular.

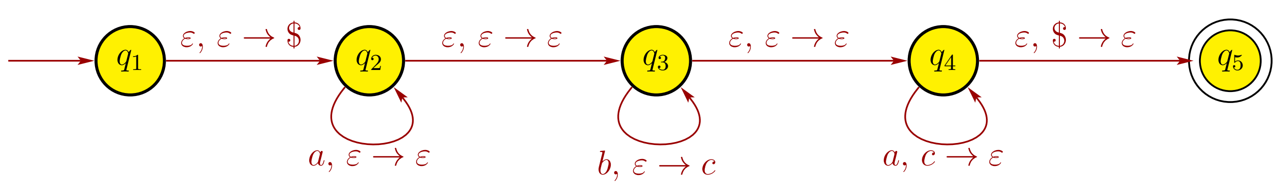
(j) TRUE FALSE — If languages A and B have context-free grammars, then $A \cap B$ must be recognized by a PDA.

2. **[20 points]** Give short answers to each of the following parts. Each answer should be at most a few sentences. Be sure to define any notation that you use.

(a) Give a regular expression for the language recognized by the NFA below.



(b) Consider the language A have the following PDA:



List the first 5 strings in A in **string order**.

(c) Suppose that we are in the process of converting a CFG G with $\Sigma = \{0, 1\}$ into Chomsky normal form. We have already applied some steps in the process, and we currently have the following CFG with variables $V = \{S_0, S, A\}$ and start variable S_0:

$$\begin{aligned} S_0 &\rightarrow S \\ S &\rightarrow S1XS0 \mid \varepsilon \\ X &\rightarrow 0SX1X0 \mid 0X1 \mid \varepsilon \end{aligned}$$

In the next step, we want to remove the ε-rule $X \rightarrow \varepsilon$. Give the CFG after carrying out just this one step.

(d) Suppose a language A_1 is generated by a context-free grammar $G_1 = (V_1, \Sigma, R_1, S)$, with $V_1 = \{S, X\}$, start variable S, terminal alphabet $\Sigma = \{a, b\}$, and rules R_1 given by

$$\begin{aligned} S &\rightarrow aXaXSb \mid baa \\ X &\rightarrow XbSaaX \mid ab \end{aligned}$$

Also, a language A_2 is generated by a context-free grammar $G_2 = (V_2, \Sigma, R_2, S)$, with $V_2 = \{S, X\}$, start variable S, terminal alphabet $\Sigma = \{a, b\}$, and rules R_2 given by

$$\begin{aligned} S &\rightarrow bXaS \mid abb \\ X &\rightarrow SbXSa \mid ba \end{aligned}$$

Give a context-free grammar G_3 for $A_2 \circ A_1$. Be sure to define G_3 as a 4-tuple, explicitly specifying each part of the 4-tuple, but do not prove the correctness of your CFG G_3.

3. **[20 points]** For the next two parts, consider the language

$$C = \{ w \in \Sigma^* \mid |w| \geq 2, \text{ second-to-last symbol of } w \text{ is } a \},$$

where $\Sigma = \{a, b\}$. For a string $w = w_1 w_2 \cdots w_{n-1} w_n$ with each $w_i \in \Sigma$ and $n \geq 2$, the second-to-last symbol of w is w_{n-1}.

(a) Give a 5-tuple description for a DFA for C. Be sure to explicit define each part of the 5-tuple $(Q, \Sigma, \delta, q_1, F)$ for your DFA for C.

(b) Give a regular expression for C.

4. [**10 points**] Consider the following language:

$$D = \{\, b^i a^j b^k \mid i, j, k \geq 0,\ j = i + k \,\}.$$

Give a context-free grammar G for D. Be sure to specify G as a 4-tuple $G = (V, \Sigma, R, S)$, explicitly defining each part of the 4-tuple.

5. **[10 points]** Recall the pumping lemma for regular languages:

Theorem: If L is a regular language, then there exists a pumping length p where, if $s \in L$ with $|s| \geq p$, then s can be split into three pieces $s = xyz$ such that (i) $xy^iz \in L$ for each $i \geq 0$, (ii) $|y| \geq 1$, and (iii) $|xy| \leq p$.

Let

$$E = \{ \, w \in \Sigma^* \mid w = w^{\mathcal{R}} \text{ and } w \text{ has odd length} \, \},$$

where $\Sigma = \{c, d\}$, and $w^{\mathcal{R}}$ denotes the reverse of a string w. Is E a regular or nonregular language? If E is regular, give a regular expression for it. If E is not regular, prove that it is a nonregular language.

Circle one: **Regular Language** **Nonregular Language**

6. [**10 points**] In Theorem 3.21 we showed that a language is Turing-recognizable if and only if some enumerator enumerates it. Specifically, let L be a language recognized by a TM M, and let $s_1, s_2, \ldots$ be a list of all strings in Σ^*. In the forward direction of the proof of Theorem 3.21, we constructed an enumerator

$E =$ "Ignore the input.

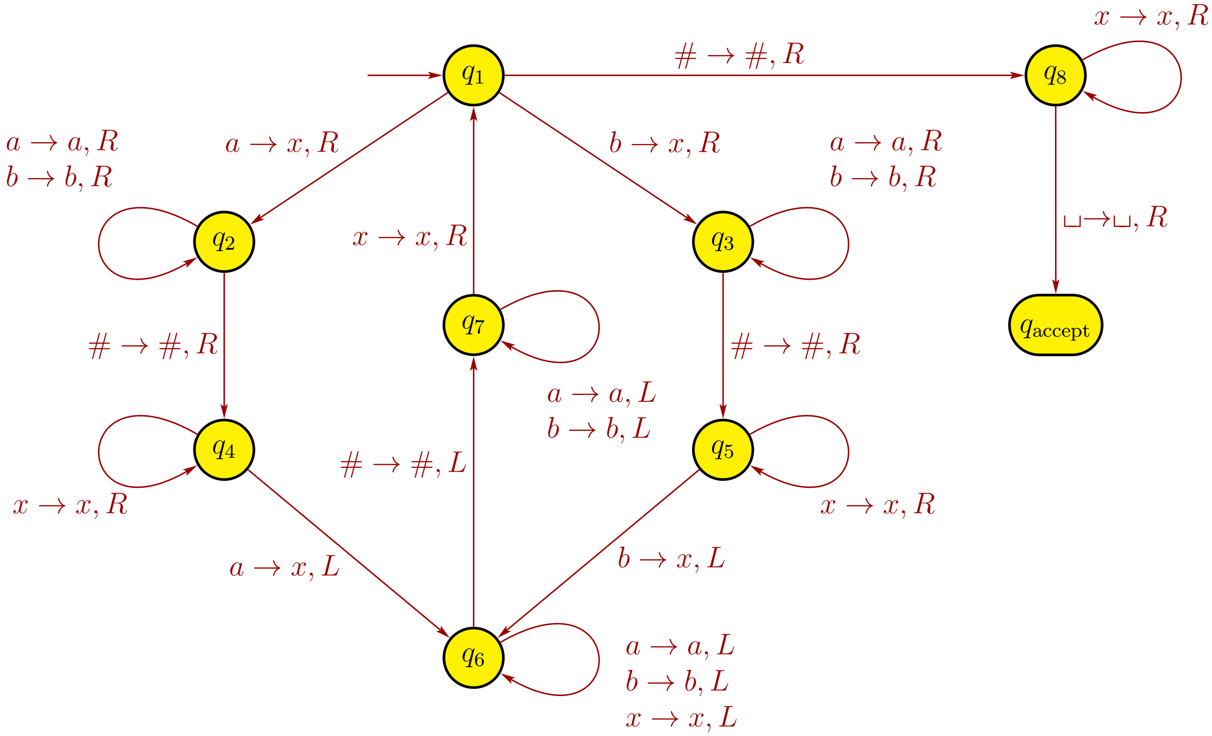
1. Repeat the following for $i = 1, 2, 3, \ldots$
2. Run M for i steps on each input $s_1, s_2, \ldots, s_i$.
3. If any computation accepts, print out corresponding string s."

Instead, for the forward direction of the proof, why didn't we construct the following simpler enumerator E'?

$E' =$ "Ignore the input.

1. Repeat the following for $i = 1, 2, 3, \ldots$
2. Run M on s_i.
3. If it accepts, print out s_i."

7. **[10 points]** Consider the below Turing machine $M = (Q, \Sigma, \Gamma, \delta, q_1, q_{\text{accept}}, q_{\text{reject}})$ with $Q = \{q_1, \ldots, q_8, q_{\text{accept}}, q_{\text{reject}}\}$, $\Sigma = \{a, b, \#\}$, $\Gamma = \{a, b, \#, x, \sqcup\}$, and transitions below.



To simplify the figure, we don't show the reject state q_{reject} or the transitions going to the reject state. Those transitions occur implicitly whenever a state lacks an outgoing transition for a particular symbol. For example, because in state q_5 no outgoing arrow with # is present, if # occurs under the head when the machine is in state q_5, it goes to state q_{reject}. For completeness, we say that in each of these transitions to the reject state, the head writes the same symbol as is read and moves right.

Give the sequence of configurations that M enters when started on input string $baba\#aab$.

If you need to enter a blank $\sqcup$ in your answer, you should instead use the plus symbol + to represent a blank.

8. **[10 points]** The next 5 questions are of type **Multiple Answers**. There can be more than one correct answer. You will earn points for every correct answer selected, and you will lose points for every wrong answer selected.

   (a) Define the following classes of languages:

   - D is the class of languages decided by Turing machines
   - F is the class of finite languages
   - G is the class of languages having context-free grammars
   - N is the class of languages recognized by NFAs
   - P is the class of languages recognized by PDAs
   - R is the class of languages having regular expressions
   - T is the class of languages recognized by Turing machines.

   Which of the following statements is true?

   i. R is a subset of F
   ii. T is closed under union
   iii. R is a subset of N
   iv. R = P
   v. D is closed under intersection
   vi. N is closed under complementation
   vii. N is a subset of R

   (b) If a language A if finite, then A must be recognized by a

   i. DFA
   ii. NFA
   iii. PDA
   iv. Turing machine
   v. k-tape Turing machine
   vi. Nondeterministic Turing machine

   (c) If a language A is context-free and B is not context-free, then

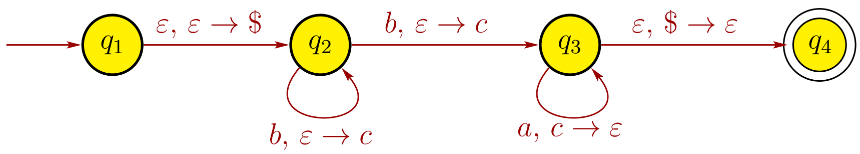
   i. $A \circ B$ must be context-free
   ii. $A \circ B$ must be non-context-free
   iii. $A \cup B$ must be context-free
   iv. $A \cup B$ must be non-context-free
   v. $A \cap B$ must be context-free
   vi. $A \cap B$ must be non-context-free
   vii. None of the other given statements is correct

   (d) The class of **languages having a regular expression** is closed under

   i. complementation
   ii. union

iii. intersection
iv. concatenation
v. Kleene star
vi. None of the given operations

(e) Consider a PDA $M = (Q, \Sigma, \Gamma, \delta, q_1, F)$, where $Q = \{q_1, q_2, q_3, q_4\}$, $\Sigma = \{a, b\}$, $\Gamma = \{a, b, c, \$\}$, $F = \{q_4\}$, and δ as specified in the following graph:



Which of the following CFGs (if any) generates the language $L(M)$? Below, only the rules of each CFG are specified.

i. $S \to aSb \mid \varepsilon$
ii. $S \to aSa \mid bSb \mid b$
iii. $S \to bSa \mid \varepsilon$
iv. $S \to bSa \mid ba$
v. $S \to bSc \mid X \mid Y$
$X \to aXb \mid S$
$Y \to cYa \mid b$
vi. $S \to bSa \mid X$
$X \to bXa \mid ba$
vii. $S \to aSb \mid ab$
viii. None of the other choices is correct.

(f) The class of **finte languages** is closed under

i. union
ii. complementation
iii. intersection
iv. concatenation
v. Kleene star
vi. None of the other answers is correct.