Midterm Exam II
CIS 341: Introduction to Logic and Automata - Fall 1996
Prof. Marvin K. Nakayama

Print Name (last name first):

Student Number:

- This exam will be 1 hour and 25 minutes in length.
- This is an open-book, open-note exam.
- For all problems, follow these instructions:

1. Show your work and give reasons (except for question 1).
2. Give only your answers in the spaces provided. Only what you put in the answer space will be graded, and points will be deducted for any scratch work in the answer space. Use the scratch-work area to work out your answers before filling in the answer space.
3. FA stands for finite automaton; TG stands for transition graph.
4. For any proofs, be sure to provide a detailed, step-by-step argument, with justifications (e.g., cite a theorem or definition in the textbook) for each step. You may assume that any theorems and results in the textbook hold; i.e., you do not have to reprove the results in the textbook for which proofs are already given. When using a theorem, definition, or result from the textbook, make sure you refer to it by number or page number (e.g., Theorem 3).

| Problem | 1 | 2 | 3 | 4 | Total |
| :---: | :--- | :--- | :--- | :--- | :---: |
| Points |  |  |  |  |  |

1. [ 30 points] For each of the following, circle TRUE if the statement is correct. Otherwise, circle FALSE
(a) TRUE FALSE - If $L_{1}$ is a regular language and $L_{2}$ is a nonregular language, then $L_{1} L_{2}$ must be a regular language.
(b) TRUE FALSE - If $L_{1}$ is a regular language and $L_{2}$ is a nonregular language, then $L_{1} L_{2}$ must be a nonregular language.
(c) TRUE FALSE - A Moore machine is nondeterministic.
(d) TRUE FALSE - A nonregular language may have only finitely many different words.
(e) TRUE FALSE - The concatenation of two regular languages is always a regular language.
(f) TRUE FALSE - Nonregular languages are languages accepted by nondeterministic finite automata.
(g) TRUE FALSE - Every Moore machine has an equivalent Mealy machine.
(h) TRUE FALSE - Any subset of any nonregular language is also a nonregular language.
(i) TRUE FALSE - Any subset of any regular language is also a regular language.
(j) TRUE FALSE - If $L$ is a regular language, then $L^{*}$ is a regular language.
2. [30 points] Let $L$ be the language exactly accepted by the following finite automaton:


Give a regular expression for $L$.

Regular Expression:

Scratch-work area
3. [ $\mathbf{2 0}$ points] Theorem 10 of the textbook states that if $L_{1}$ and $L_{2}$ are regular languages, then $L_{1}+L_{2}$ is a regular language. This result can then be applied to show that the union of any finite number of regular languages is also a regular language; i.e., if $L_{1}, L_{2}, \ldots, L_{m}$, $m<\infty$, are regular languages, then $L_{1}+L_{2}+\cdots+L_{m}$ is a regular language. Now suppose that we have an infinite collection of regular languages $L_{1}, L_{2}, L_{3}, \ldots$. Is it necessarily the case that $L_{1}+L_{2}+L_{3}+\cdots$ is a regular language? In other words, is the union of any infinite collection of regular languages always a regular language?

## YES NO (Circle one)

If your answer is YES, give a proof. If your answer is NO, give a counterexample. Explain your answer.
4. [20 points] Suppose $L_{1}$ and $L_{2}$ are languages over an alphabet $\Sigma$. Assume that there exists some nondeterministic finite automaton that exactly accepts $L_{1}$, and that $L_{2}$ has only finitely many words. Prove that $L_{1} \cap L_{2}$ is a regular language.

