Midterm Exam I
CIS 341: Introduction to Logic and Automata - Fall 1999
Prof. Marvin K. Nakayama

Print Name (last name first):

Student Number: $\qquad$

- This exam will be 1 hour and 25 minutes in length.
- This is a closed-book, closed-note exam.
- For all problems, follow these instructions:

1. Give only your answers in the spaces provided. I will only grade what you put in the answer space, and I will take off points for any scratch work in the answer space. Use the scratch-work area to work out your answers before filling in the answer space.
2. FA stands for finite automaton; TG stands for transition graph.
3. For any proofs, be sure to provide a step-by-step argument, with justifications for every step.

| Problem | 1 | 2 | 3 | 4 | 5 | Total |
| :---: | :--- | :--- | :--- | :--- | :--- | :---: |
| Points |  |  |  |  |  |  |

1. [20 points] For each of the following, circle TRUE if the statement is correct. Otherwise, circle FALSE
(a) TRUE FALSE - If $L$ is a finite language, then it has a regular expression.
(b) TRUE FALSE - If $L$ is a finite language, then $L^{*}$ must be infinite.
(c) TRUE FALSE - All transitions graphs are non-deterministic.
(d) TRUE FALSE - A finite automaton may have more than one initial state.
(e) TRUE FALSE - A finite automaton may have more than one final state.
(f) TRUE FALSE - If $L$ is an infinite language, then $L^{*}$ must be infinite.
(g) TRUE FALSE - If $L$ is any language, then $L^{*}=L^{+}$.
(h) TRUE FALSE - If $L$ is any language, then $L^{++}=L^{+}$.
(i) TRUE FALSE - The regular expression $\mathbf{b}^{*}(\mathbf{a a})^{*} \mathbf{b}^{*}$ generates the language over the alphabet $\{a, b\}$ consisting of all words having an even number of $a$ 's.
(j) TRUE FALSE - A finite automaton may have no final states.
2. [20 points] For each of the following languages $L$ over the alphabet $\Sigma=\{a, b\}$, give a regular expression for $L$.
(a) $L$ exactly consists of all words that begin with $a$ and end with $b a$.

## Regular Expression:

(b) Lexactly consists of all words that have an even number of $a$ 's and an even number of $b$ 's.

Regular Expression:

## Scratch-work area

3. [20 points] For each of the following languages $L$ over the alphabet $\Sigma=\{a, b\}$, give a finite automaton that accepts exactly $L$.
(a) $L$ exactly consists of all words that have an even number of $a$ 's and an odd number of $b$ 's.

## Draw finite automaton here:

(b) $L$ exactly consists of all words whose second-to-last letter is $b$.

Draw finite automaton here:

## Scratch-work area

4. [20 points] In class we discussed the language PALINDROME over the alphabet $\Sigma=$ $\{a, b\}$.
(a) Give a definition of the language PALINDROME. Don't just list out the words.
(b) List all words in PALINDROME having length 3.
(c) List all words in PALINDROME having length 4.
(d) How many words in PALINDROME have length exactly $n$ ? Be sure to give an explanation.

## Scratch-work area

5. [20 points] Let $S$ and $T$ be any arbitrary sets of strings. If $S^{*}=T^{*}$, is it always the case that $S=T$ ?
YES NO (Circle one)

If your answer is YES, give a proof. If your answer is NO, give a counterexample. Explain your answer.

