

Midterm Exam II  
CIS 341: Introduction to Logic and Automata — Fall 1999  
Prof. Marvin K. Nakayama

Print Name (last name first): \_\_\_\_\_

Student Number: \_\_\_\_\_

I have read and understand all of the instructions below, and I will obey the Academic Honor Code.

\_\_\_\_\_  
Signature and Date

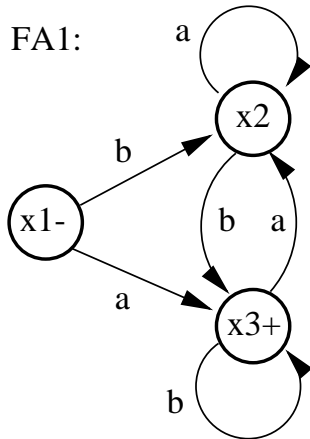
- This exam will be 1 hour and 25 minutes in length.
- This is an closed-book, closed-note exam.
- For all problems, follow these instructions:
  1. Show your work and give reasons (except for question 1).
  2. Give only your answers in the spaces provided. Only what you put in the answer space will be graded, and points will be deducted for any scratch work in the answer space. Use the scratch-work area to work out your answers before filling in the answer space.
  3. FA stands for finite automaton; TG stands for transition graph.
  4. For any proofs, be sure to provide a detailed, step-by-step argument, with justifications for each step. You may assume that any theorems in the textbook hold; i.e., you do not have to reprove the theorems in the textbook. If you use a theorem, definition, or result from the textbook in your proofs, be sure to give enough details so that I know what you are referring to; e.g., say something like “Using the theorem that states that  $S^{**} = S^*$ , we have that ...”.

Problem	1	2	3	4	Total
Points					

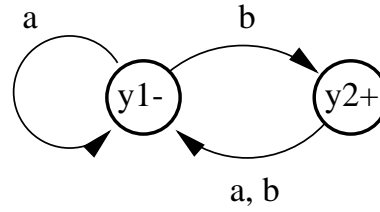
1. [30 points] For each of the following, circle TRUE if the statement is correct. Otherwise, circle FALSE

- (a) TRUE FALSE — If a Moore machine and a Mealy machine are equivalent, then they print out exactly the same output on every possible input string.
- (b) TRUE FALSE — If  $L$  is a regular language, then  $L^*$  is a regular language.
- (c) TRUE FALSE — If  $L_1$  and  $L_2$  are two language such that  $L_1 \cap L_2' = \emptyset$ , then  $L_1 = L_2$ .
- (d) TRUE FALSE — All finite automata are also transition graphs.
- (e) TRUE FALSE — If  $L_1$  and  $L_2$  are regular languages, then  $L_1L_2$  may be a nonregular language.
- (f) TRUE FALSE — If  $L$  is a regular language, then  $L$  has a transition graph.
- (g) TRUE FALSE — If  $L$  is a nonregular language, then there is some nondeterministic finite automaton that accepts  $L$ .
- (h) TRUE FALSE — An effective procedure to decide if a given finite automaton accepts any words is to test all possible strings on the FA and see if any are accepted.
- (i) TRUE FALSE — All transition graphs are also finite automata.
- (j) TRUE FALSE — All nondeterministic finite automata are nondeterministic.

2. [25 points] Let  $L_1$  and  $L_2$  be the languages accepted by the following finite automata:



FA2:



Build an FA that will accept exactly the language  $L_1 \cap L_2$ .

**Draw finite automaton here:**

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**Scratch-work area**

3. [20 points] Let  $L_1$  be a regular language and let  $L_2$  be a finite language.

(a) Prove that  $L_1 + L_2$  is a regular language.

(b) Prove that  $L_1 - L_2$  is a regular language. Remark:  $L_1 - L_2 = \{w : w \in L_1 \text{ and } w \notin L_2\}$ .

4. [25 points] Recall that the pumping lemma says:

Let  $L$  be an infinite language accepted by a finite automaton with  $N$  states. If  $w \in L$  with  $\text{length}(w) > N$ , then there exist strings  $x$ ,  $y$ , and  $z$  such that

- (i)  $w = xyz$
- (ii)  $y \neq \Lambda$
- (iii)  $\text{length}(x) + \text{length}(y) \leq N$
- (iv)  $w = xy^kz \in L$  for all  $k = 0, 1, 2, \dots$

Use the pumping lemma to prove that the language  $\{a^m b^m a^m : m = 0, 1, 2, \dots\}$  is nonregular.