CS 341-H01, Fall 2026, Face-to-Face Section
Solutions for Quiz 1

1. Multiple choice.

1.1. Answer: (b).

- Corollary 3.18 establishes that a language L is Turing-recognizable if and only if a nondeterministic TM recognizes L, making (b) correct.
- Choice (a) is incorrect. For example, the language $A = \{ s\#s \mid s \in \{0,1\}^* \}$ is not regular, which can be shown by slightly modifying the proof (slide 1-107) that the language $\{ ww \mid w \in \{0,1\}^* \}$ is nonregular; e.g., use string $0^p\#0^p$ in the pumping lemma for regular languages to get a contradiction. Thus, because A is nonregular, A does not have an NFA by Kleene's Theorem. But slide 3-9 gives a TM that recognizes (and actually decides) A, so (a) is incorrect. This same language is infinite, so (c) is also incorrect.
- For the alphabet $\Sigma = \{0,1\}$, consider the language $A = \emptyset$, which is finite. We can design a TM $M = (Q, \Sigma, \Gamma, \delta, q_0, q_{\text{accept}}, q_{\text{reject}})$ that recognizes A by having $q_{\text{reject}} = q_0$, so $L(M) = \emptyset$, making (d) incorrect.

1.2. Answer: (a).

- Theorem 4.4 shows that E_{DFA} is decidable, so (a) is correct, and (b) is incorrect.
- There are infinitely many DFAs that recognize the empty language. For example, any DFA that has no accept states recognizes $\emptyset$, and there are infinitely many such DFAs. Thus, (c) is incorrect.

1.3. Answer: (g).

- As noted on slide 4-13, languages A and B are equal if and only if their symmetric difference $(A \cap \overline{B}) \cup (\overline{A} \cap B)$ is empty. Thus, $A \neq B$ if and only if $(A \cap \overline{B}) \cup (\overline{A} \cap B) \neq \emptyset$, which is equivalent to $(A \cap \overline{B}) \neq \emptyset$ or $(\overline{A} \cap B) \neq \emptyset$, so (g) is correct.
- Suppose $\Sigma = \{0\}$, and we will give examples of languages A and B over Σ with $A \neq B$ to show the other choices are incorrect.
  (a) Let $A = \{\varepsilon\}$ and $B = \{0\}$. Then $A \cup B = \{\varepsilon, 0\}$, so $\overline{A \cup B} = \{ 0^n \mid n \geq 2 \} \neq \emptyset$, making (a) incorrect.
  (b) If $A = \Sigma^*$ and $B = \{0\}$, then $\overline{A} = \emptyset$ and $\overline{A} \cup B = \{0\} \neq \emptyset$, so (b) is incorrect.
  (c) Suppose that $A = \{0, 00\}$ and $B = \{0\}$. Then $A \cap B = \{0\}$, and $\overline{A \cap B} = \Sigma^* - \{0\} = \{ 0^n \mid n \geq 1 \} \neq \emptyset$, so (c) is incorrect.
  (d) If $\overline{A} \cap B = \emptyset$ and $A \cap \overline{B} = \emptyset$, then $(\overline{A} \cap B) \cup (A \cap \overline{B}) = \emptyset$, making $A = B$ because their symmetric difference is empty. Thus, we cannot have $A \neq B$, so (d) is incorrect.
  (e) Let $A = \{\varepsilon\}$ and $B = \{0\}$. Then $\overline{A} = \{ 0^n \mid n \geq 1 \}$, and $\overline{A} \cap B = \{0\} \neq \emptyset$. Also, $\overline{B} = \{ 0^n \mid n \neq 1 \}$, and $A \cap \overline{B} = \{\varepsilon\} \neq \emptyset$, making (e) incorrect.

(f) Let $A = \{\varepsilon\}$ and $B = \Sigma^*$. Then $\overline{B} = \emptyset$ and $A \cap \overline{B} = \emptyset$, so (f) is incorrect.

1.4. Answer: (c).

- Choice (c) is correct by Theorem 3.21. Choice (a) is incorrect by Theorem 3.13. Choice (b) is inccorect by Theorem 3.16.

1.5. Answer: (d).

- If a TM recognizes A, it must accept each string $w \in A$, and must reject or loop on each string $w \notin A$.

2. **[15 points]** Show that the class of Turing-recognizable languages is closed under union. Explain your answer.

Answer: (This is HW 7, problem 2b.) For any two Turing-recognizable languages L_1 and L_2, let M_1 and M_2, respectively, be TMs that recognize them. We construct a TM M' that recognizes the union $L_1 \cup L_2$:

M' = "On input string w:

1. Run M_1 and M_2 alternately on w, one step at a time.
If either accepts, *accept*. If both halt and reject, *reject*.

To see why M' recognizes $L_1 \cup L_2$, first consider $w \in L_1 \cup L_2$. Then w is in L_1 or in L_2 (or both). If $w \in L_1$, then M_1 accepts w, so M' will eventually accept w. Similarly, if $w \in L_2$, then M_2 accepts w, so M' will eventually accept w. On the other hand, if $w \notin L_1 \cup L_2$, then $w \notin L_1$ and $w \notin L_2$. Thus, neither M_1 nor M_2 accepts w, so M' will also not accept w. Hence, M' recognizes $L_1 \cup L_2$. Note that if neither M_1 nor M_2 accepts w and one of them does so by looping, then M' will loop, but this is fine because we only needed M' to *recognize* and not *decide* $L_1 \cup L_2$.

3. **[15 points]** Consider the problem of determining **whether an NFA and regular expression are equivalent**. Express this problem as a language and show that it is decidable.

Answer: Define the language as

$$C = \{\langle N, R\rangle \mid N \text{ is an NFA and } R \text{ is a regular expression with } L(N) = L(R)\}.$$

Recall that the proof of Theorem 4.5 defines a Turing machine F that decides the language $EQ_{\text{DFA}} = \{\langle A, B\rangle \mid A \text{ and } B \text{ are DFAs and } L(A) = L(B)\}$. Then the following Turing machine T decides C:

T = "On input $\langle N, R\rangle$, where N is an NFA and R is a regular expression:

1. Convert N into an equivalent DFA D
using the algorithm in the proof of Kleene's Theorem.

2. Convert R into an equivalent DFA D'
using the algorithm in the proof of Kleene's Theorem.

3. Run TM F for deciding EQ_{DFA} (from Theorem 4.5) on input $\langle D, D'\rangle$.

4. If F accepts, *accept*. If F rejects, *reject*."