Lecture Notes 7
Basics of Arena and TikeTown

- John Bungee's Problem at TikeTown
  - Outline of Process
- Simulation Model using Spreadsheets or Standard Programming Languages
  - Event-driven versus Process-Oriented Simulation
- Design of a Simulation Study
  - Goals of the Study
  - Components of the Process-Oriented Simulation Model
- ARENA Simulation Model
- Quick Primer on Queueing Theory
- Be sure to check course webpage for reading assignment and HW

Big Picture

Business and Engineering Processes:

- Manufacturing and service **operations are process-oriented**.
  - *Unfinished products* must undergo one or more *processes* before they are *finished*.
- Want to improve the performance of their processes in order to provide superior products to their customers.
- Simulation modeling is a brute-force tool for *understanding* and *improving* existing processes and *designing* new ones.
- The key block for most process-oriented simulation models is the *simple queue*.

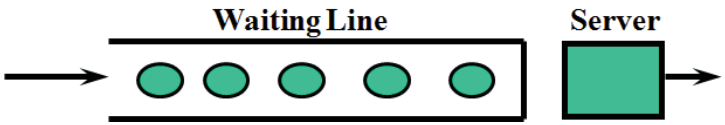
Objective

- Simulation of a simple queueing system using ARENA
- Introduction to the *process-oriented* simulation modeling

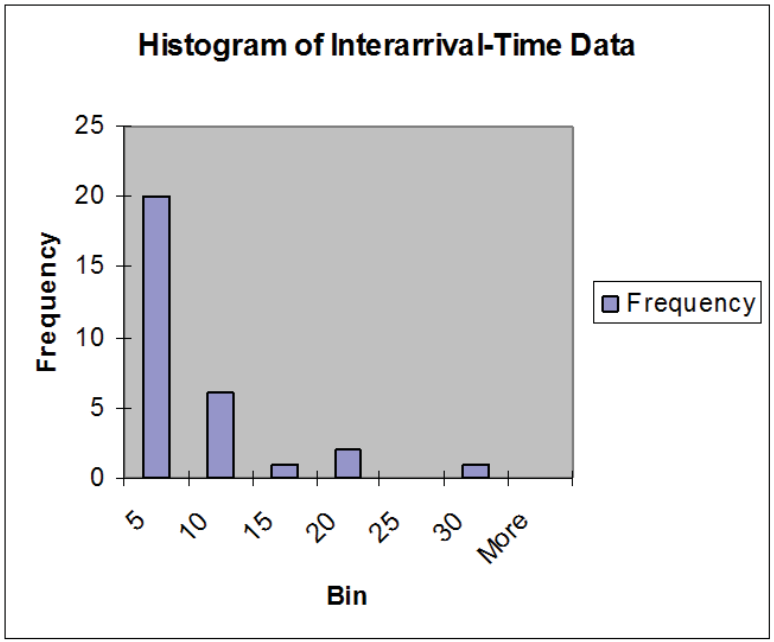
Problem Description

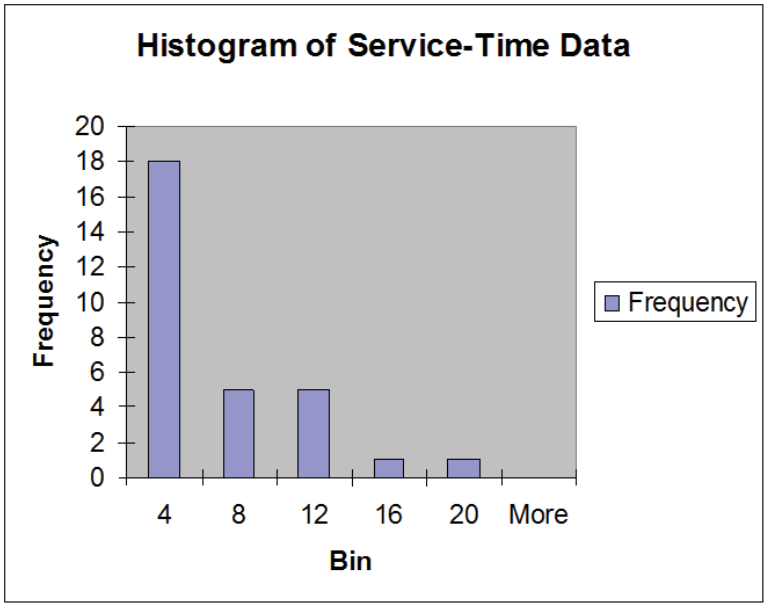
- **TikeTown**
  - Small chain of discount stores for kids stuff
  - Mission: to "provide low prices and excellent service"
- Problem:
  - Service at check-out counter below standard.
- Puzzle:
  - Consultant did a detailed simulation-based analysis of proposed service-counter design.
  - His conclusion was that one counter would be enough.
  - But the length of the waiting lines attests to the contrary.
- Our task:
  - To find out what went wrong.

The Process



- **Problem:** Determine a probabilistic model for the distribution of the interarrival and service times.
- How were the distributions determined in the case?
- Suggest an alternative approach.





Simulation Model using Spreadsheets or Standard Programming Language

1. Generate interarrival times for 1000 customers

   A_n = interarrival time between customers n and $n+1$

2. Generate service times for 1000 customers

   S_n = service time for customer n

3. Compute W_{n+1}, waiting time for customer $n+1$, using **Lindley's Equation**:

   Waiting Time for Next Customer
   = max(0, Waiting time for Current Customer
   + Service Time for Current Customer
   − Interarrival time between current and next customers)

Lindley's Equation

- $W_{n+1} = \max(0,\ W_n + S_n - A_n)$
  - W_i = waiting of ith customer; $W_1 = 0$
  - S_n = service time of nth customer
  - A_n = interarrival time between nth and $(n+1)$th customers
- Customer n arrives at time $T_n = T_{n-1} + A_{n-1}$
  - Departure time is $D_n = T_n + W_n + S_n$
- If customer $n+1$ in queue when customer n departs, then customer $n+1$ starts service at time D_n.

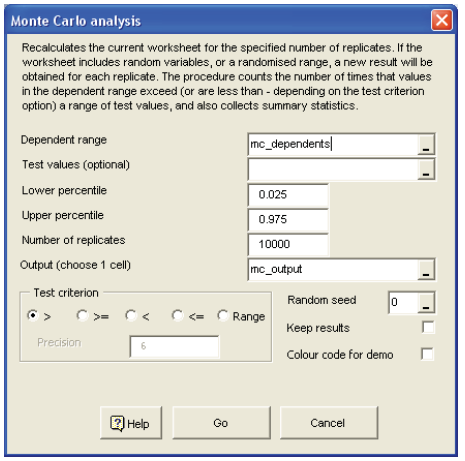
Lindley's Equation

- Recall: customer n **departs** at time $D_n = T_n + W_n + S_n$.
- Customer $n+1$ **arrives** at time $T_{n+1} = T_n + A_n$
  - If $T_{n+1} \geq D_n$, then customer $n+1$ does not wait, so
    $$W_{n+1} = 0 \quad \text{when} \quad D_n - T_{n+1} \leq 0$$
  - If $T_{n+1} < D_n$, then customer $n+1$ has a wait of
    $$W_{n+1} = D_n - T_{n+1} \quad \text{when} \quad D_n - T_{n+1} > 0$$
- Combining the two cases gives
$$\begin{aligned} W_{n+1} &= \max(D_n - T_{n+1}, 0) \\ &= \max(W_n + S_n + T_n - T_{n+1}, 0) \\ &= \max(W_n + S_n - A_n, 0) \end{aligned}$$
since $T_{n+1} - T_n = A_n$.

Independent Replications Using Excel/PopTools

PopTools: free add-in for 32-bit Excel in Windows, available from web.

- After installing in Excel and building model, select: `Add-Ins > PopTools > Simulation tools > Monte Carlo Analysis`



Independent Replications Using Excel/PopTools

- **Dependent range**: cell(s) with model's *output* from replication.
  - If more than 1 cell, must be in a column.
- **Output (choose 1 cell)**: where PopTools puts *computed stats*.
  - If d outputs, then computed stats appear in $(d+1) \times 6$ matrix.
- **Number of replicates**: specify sample size $n = \#$ indep reps.
- Construct CI using *Mean* and *Variance* from computed stats.
- **Keep results**: *check* to save outputs from all reps in new worksheet.
- **Lower percentile** of p returns (np)th smallest output value from all n reps.
  - **Upper percentile** of q returns (nq)th smallest output value.
  - Computed **Lower CL** and **Upper CL** give a *tolerance interval*, not a CI!

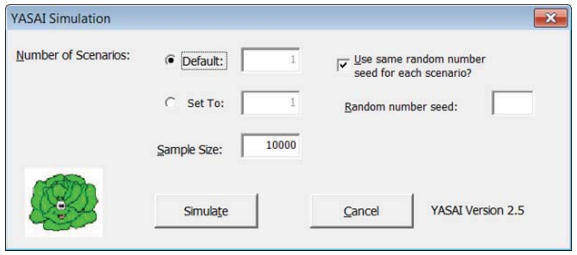
Independent Replications Using Excel/Yasai

Yasai: free add-in for 32- and 64-bit Excel for Windows and Mac.

- Download from web and install.
- Use formula SIMOUTPUT(*x*,*name*) in new cell to specify output variable of model.
  - *x* is address of one cell or formula for simulation output.
  - *name* is user-specified name of output variable in double quotes.
  - **Example:** =SIMOUTPUT(A4+B7,"profit") defines output variable named "profit" with value A4+B7.
- Each output variable needs separate SIMOUTPUT in different cell.

Independent Replications Using Excel/Yasai

- To run independent reps, in Excel, select:
  `Add-Ins > YASAI > Simulation...`



- After all runs done, *computed stats* in new worksheet **Simulation Output 1**
- To get outputs from all reps **in sorted order**,
  `Add-Ins > YASAI > Charts...`
  - Select *Variable*, and specify *Chart Type* as *Cumulative Plot*.

Event vs. Process Orientation

- **Key Observation:** Spreadsheet model centered around
  - Customer arrivals
  - Customer departures
  - Customer service initiation
- **Problem:** Complicated for very complex processes.
- **Solution:** Process orientation
  - Put yourself in the position of a typical customer
    - ▷ Customer arrives and joins the queue
    - ▷ Wait until you reach front of queue and counter is free.
    - ▷ Start service
    - ▷ Complete service
    - ▷ Leave

Design of Simulation Study

- **Goal of the Study**
  - Predict flow times and queue length for current counter design.
- **Performance Measures**
  - *Average waiting time in system*
    - ▷ Discrete-time Statistic or Tally
  - *Average flow time through system*
    - ▷ Another Discrete-time Statistic or Tally
  - *Time-average queue length*
    - ▷ Time-Persistent Statistic
  - *Server Utilization*
    - ▷ Another Time-Persistent Statistic

Components of Simulation Model

- **Entities**
  - The dynamic objects of the simulation model.
  - They move around, change status, affect and are affected by other entities.
  - Most entities represent *real* things in a simulation.
  - But sometimes we have *fake* entities (the breakdown demon)
  - **TikeTown Example:** Entities are the customers

Components of Simulation Model

- **Attributes** (think of as local variables)
  - Individualized characteristics of entities
    - ▷ Due Date
    - ▷ Priority
  - Attributes can be static or dynamic
- **Global Variables** (called just *Variables* in ARENA)
  - A piece of information that reflects some characteristics of the *whole system*.
  - Built-in Variables:
    - ▷ Number in queue
    - ▷ Number of busy resources
  - User-Defined Variables:
    - ▷ Shift

Components of Simulation Model

- **Resources**
  - Entities compete with each other for service from resources.
  - Examples: personnel, service counters, etc.
  - Entities **seize** a resource when available, and **release** it when finished.
  - A resource can have more than one unit of **capacity** (e.g., multi-server systems)
  - Our Example:

    Resource = Counter Server with capacity 1

Additional Components of Simulation Model

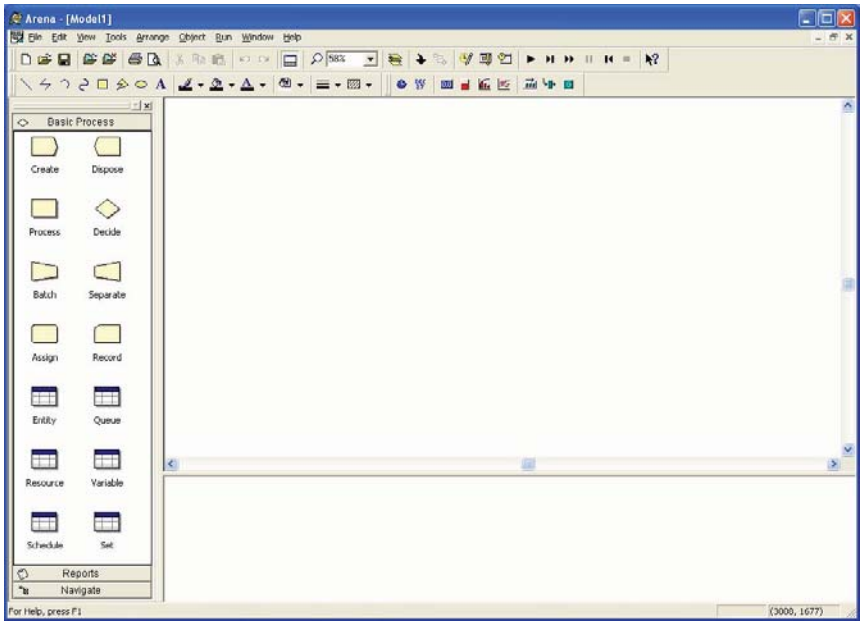
- **Queue**
  - Waiting place for entities that cannot seize a resource.
  - Queues can have finite or infinite capacity.
- **Statistical Accumulators**
  - Keeps track of output performance measures.
  - Our Example:
    - ▷ Waiting Time
    - ▷ Flow Time
    - ▷ Queue Length
    - ▷ Server Utilization
  - ARENA defines these and automatically updates them.

Additional Component of Simulation Model: Event

- **Definition:** An **event** occurs at a simulated time and can change any or all of the following:
  - Attribute
  - Variables
  - Statistical Accumulators
- Execution of the simulation
  - **Event calendar** keeps track of all known future events.
  - Each record in the calendar specifies the event type and the attribute(s) that will be affected by that event
  - Future events are sorted in chronological order.

ARENA Tutorial: Build Model for TikeTown Case

- ARENA is only for Windows.
- When installing ARENA, be sure to accept the default location.
- Open ARENA using:
  Start/Programs/Rockwell Software/Arena/Arena
- To create a new model file
  - Click on *New* icon (first icon from left)
  - Or use menu *File/New*.



ARENA Tutorial: cont.

On the left of the screen are the **templates**.

- The **Basic Process** template panel is loaded by default.
  - This template has the **modules** needed to build simple simulation models.
- ARENA has several other template panels that can help you develop more complicated simulation models.
  - To attach other template panels, use `File/Template Panel/Attach...`
- Each template panel contains a variety of modeling modules that serve several purposes.

ARENA Tutorial: cont.

TikeTown model uses 5 modules from Basic Process template panel:

- **Create**: generates new customers.
- **Process**: represents the service process.
- **Dispose**: removes customers from the model.
- **Entity**: used to define properties of entities via spreadsheet.
- **Resource**: used to define properties of resources via spreadsheet.

ARENA Tutorial: Create Module

- Birth node for all new arrivals.
- To place **Create** module
  - Click on Create icon from Basic Process Panel
  - Drag it to the location in model workspace where you want to place module.
- To enter Create info
  - Double click on just-placed polygon labeled **Create 1** to open it.
  - Dialog box opens up.

ARENA Tutorial: Create Module

Dialog Box:

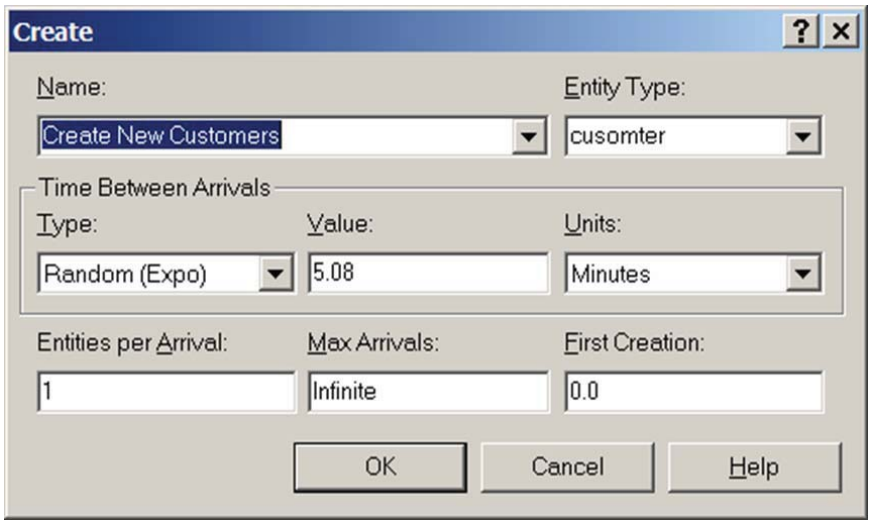
- **Name**: Specify the name of the module
- **Entity Type**: Specify name of type of entity
- **Time Between Arrivals**:
  - **Type**: Specify the process for how entities arrive; often choose *Random(Expo)*.
  - If *Random(Expo)* chosen, then exponentially distributed times
    - ▷ **Value**: to specify mean interarrival time.
    - ▷ **Units**: to specify time units.
  - **Entities per Arrival**: can allow for batch arrivals.
  - **Max Arrivals**: specify total # entities to create
  - **First Creation**: time first entity is created.

ARENA Tutorial: Create Module

For TikeTown model:

- **Name**: Create New Customers
- **Entity Type**: customer
- **Time Between Arrivals**:
  - **Type**: Random(Expo)
  - **Value**: 5.08
  - **Units**: Minutes
- **Entities per Arrival**: 1
- **Max Arrivals**: Infinite
- **First Creation**: 0.0

ARENA Tutorial: Create Module



ARENA Tutorial: Process Module

- We will use this to specify service counter (both resource and queue)
- To place the **Process module**
  - Click on Process icon from the Basic Process template panel.
  - Drag it to the model workspace.
- To enter server data
  - Double click on the just-placed box labeled **Process 1** to open it.
  - New dialog box opens up

ARENA Tutorial: Process Module

Dialog Box:

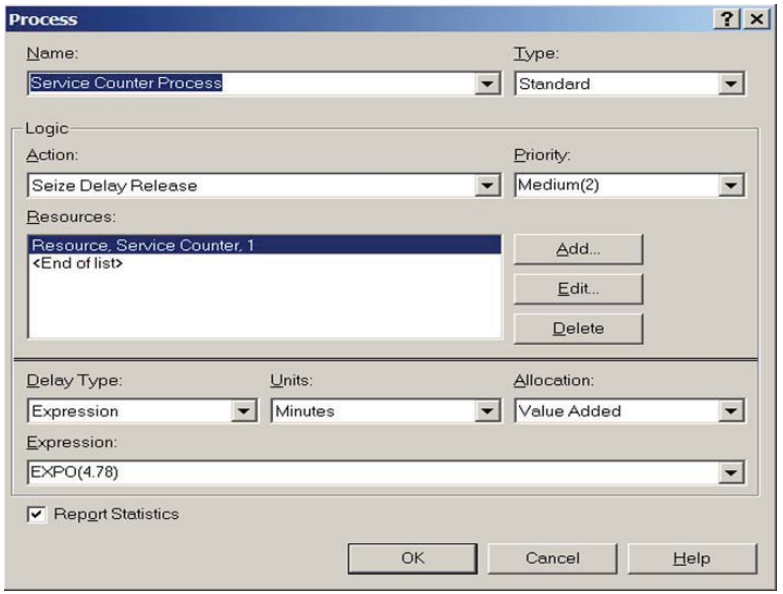
- **Name**: Specify the name of the module
- **Type**: Either standard or submodel
- **Logic**:
  - **Action**: Specify what entities do at *resources* listed for this module
  - **Priority**: specify integer for priority (lower # = higher priority)
  - **Resources**: specify number of units of the resource needed to process any entity coming to this module.
- **Delay Type**: specify distribution or expression for processing time
- **Units**: specify time units of delay
- If *Delay Type* specified to be *Expression*, then use Expression box to specify expression for delay.

ARENA Tutorial: Process Module

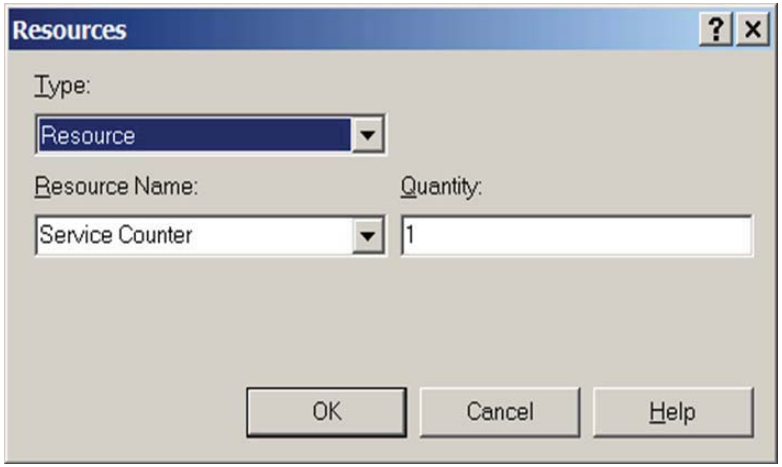
For TikeTown model:

- **Name**: Service Counter Process
- **Type**: Standard
- **Logic**:
  - **Action**: Seize, Delay, Release
  - **Priority**: Medium(2)
  - **Resources**: Click on Add button. In new Resource window,
    - ▷ **Type**: Resource
    - ▷ **Resource Name**: Service Counter
    - ▷ **Quantity**: 1
- **Delay Type**: Expression
- **Units**: Minutes
- **Expression**: EXPO(4.78)

ARENA Tutorial: Process Module



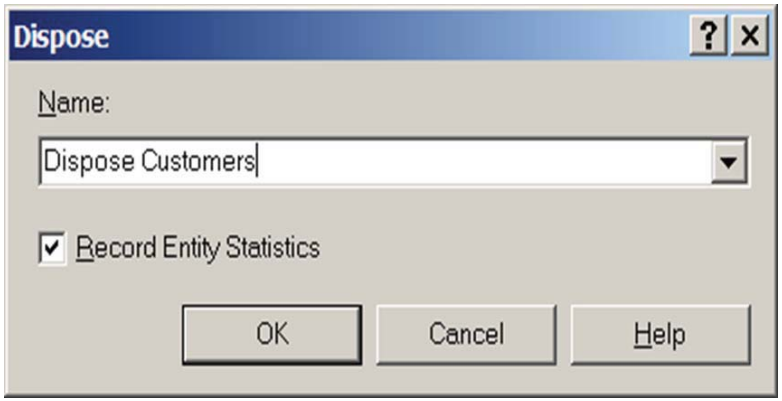
ARENA Tutorial: After Clicking on Add. . . Button



ARENA Tutorial: Dispose Module

- **Dispose Module**
  - Removes entity from model
  - Collect statistics about leaving entities
- Place the Dispose module.
  - Click on the Dispose icon from the Basic Process panel.
  - Drag it to the model workspace.
- To enter Dispose data
  - Double-click polygon labeled **Dispose 1**
  - New dialog box opens up
  - **Name**: Dispose Customers
  - Make sure check next to **Record Entity Statistics**.
    - ▷ This tells Arena to collect *flowtimes*, etc., of entities.

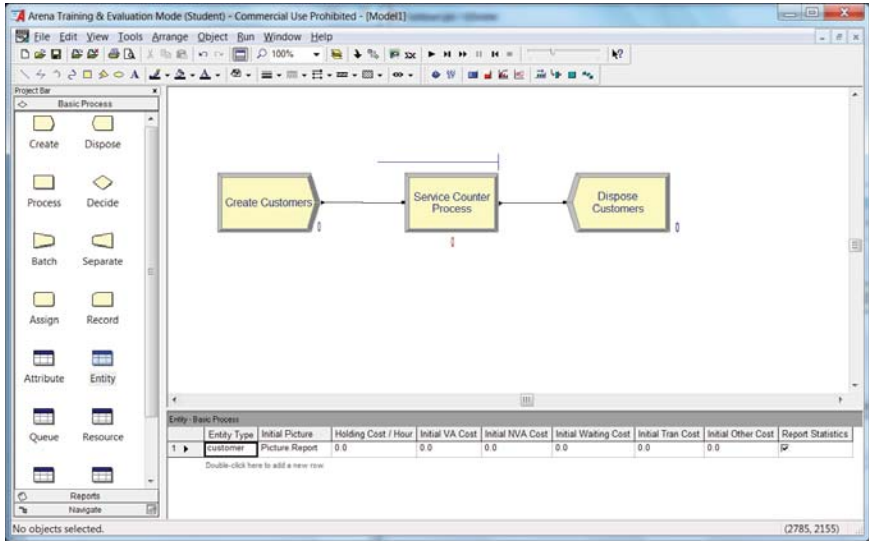
ARENA Tutorial: Dispose Module



ARENA Tutorial: Entity Module

- When you click on **Entity module** in Basic Process template panel, a spreadsheet appears in bottom right of the screen.
- Use spreadsheet to specify properties of entities.
  - Can change picture of entity used in animation.

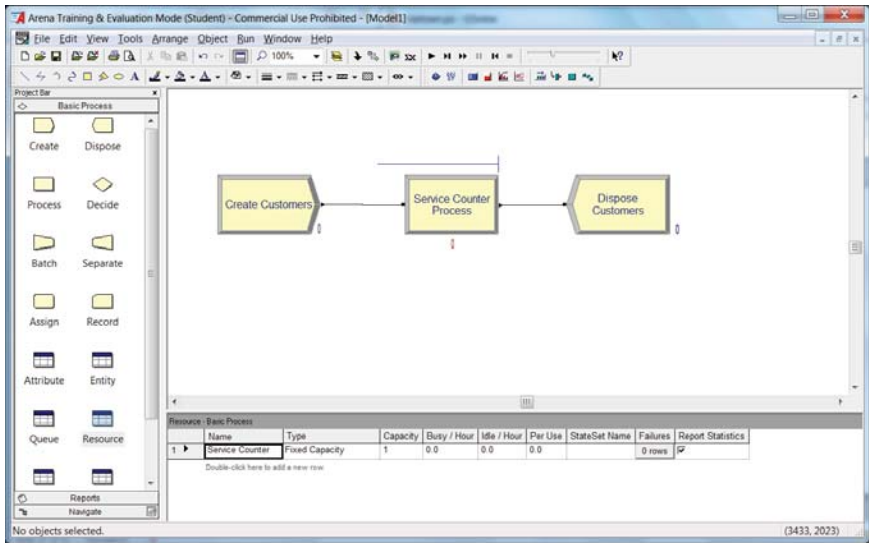
ARENA Tutorial: Entity Module



ARENA Tutorial: Resource Module

- When you click on **Resource module** in Basic Process template panel, a spreadsheet appears in bottom right of screen.
- Use spreadsheet to specify **properties** of each resource.
  - Can specify **capacity** of resource
    - ▷ e.g., multiple servers at a service counter.
  - Can specify **schedules** for workers at the resource
    - ▷ e.g., 3 servers in the first hour, 5 servers in the second hour, etc.

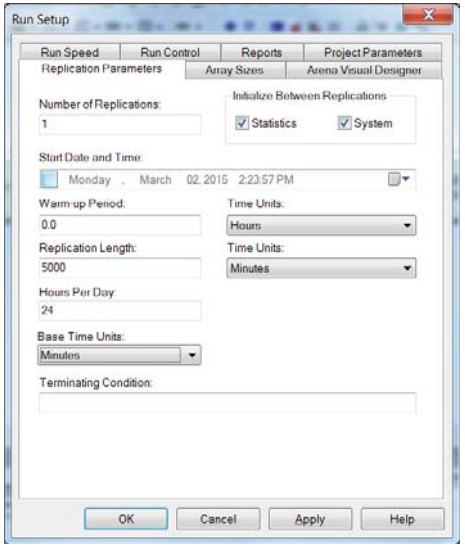
ARENA Tutorial: Resource Module



ARENA Tutorial: Run Setup

- Specify how long the simulation should run.
- Under the **Run** menu, choose **Setup. . .**
- In the Run Setup window that appears, choose **Replication Parameters** tab
  - **Replication Length**: 5000
    - ▷ Length of simulation run (in model's time units, not CPU time).
  - **Time Units**: Minutes
    - ▷ Time units for replication length
  - **Base Time Units**: Minutes
    - ▷ Time units for statistics, etc.
- You must save the file before running.

ARENA Tutorial: Run Setup



ARENA Tutorial: Module Connections

Two options:

- **Connect**: Immediate transfer
- **Route**: Includes a possible delay.
  - We may cover this later in semester.

ARENA Tutorial: Module Connections

Connect: Immediate transfer

- Click on the **connect button** 
- This turns the mouse cursor into *crosshair* $+$.
- Then, position the crosshair
  - first on the exit point from the source module (the little black triangle on its right)
  - and next on the entry point of the destination module (the little black square on its left).
- This connects the two modules.
- Repeat to complete all connections.
- Connections represent physical flow of entities in processing system.

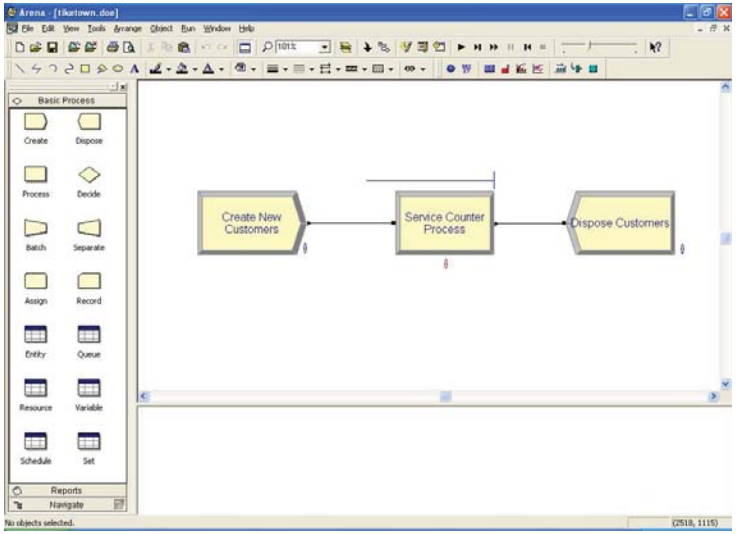
ARENA Tutorial: Running the Model

- To run the model
  - Click on the **Go** button, which is the black triangle on the upper right, and watch the simulation run.



- Buttons are **Go**, **Step** (execute only one event), **Fast-Forward**, **Pause**, **Start Over**, **End**
- To modify model after running it
  - First click on **End** button, which is the black square on right.
  - This ends current simulation run, and now model can be modified.

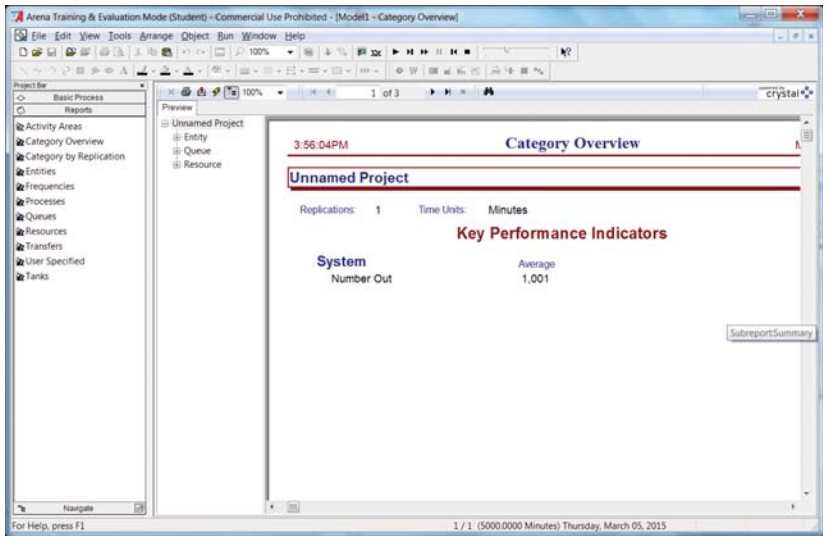
ARENA Tutorial: Running the Model



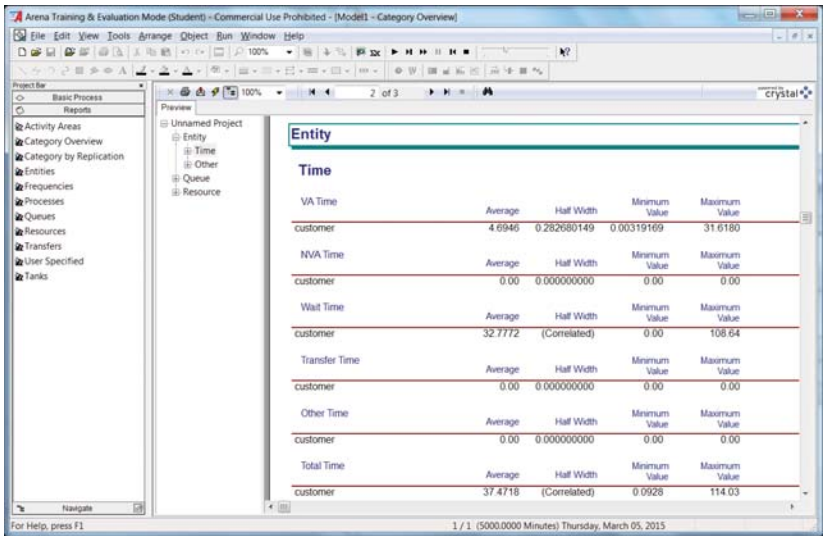
ARENA Tutorial: Remarks

- Sometimes I get an incomprehensible error message in ARENA when I try to run a model that I know is correct.
  - I'm not sure why this occurs.
  - To fix the problem, I exit out of ARENA (and possibly reboot) and start it again. The model then usually works.
- Make sure you don't put extra spaces when typing in a name of a module.
  - ARENA might give an error when checking/running.

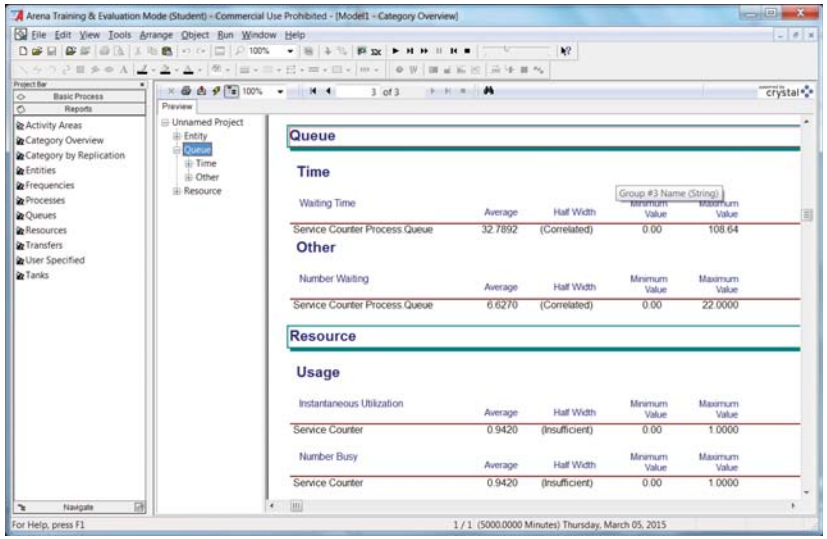
ARENA Tutorial: Simulation Results



ARENA Tutorial: Simulation Results



ARENA Tutorial: Simulation Results



ARENA Tutorial: Simulation Results

- Compare results using
  - uniform distributions, as in the TikeTown case
  - exponential distributions.
- Estimates (with 95% confidence intervals) based on 100 replications.

Distributions	Queue Length	Queueing Time
Uniform	0.6 (0.03)	3.2 (0.2)
Exponential	11.2 (1.5)	56.4 (7.0)

- Why is the mean waiting time so different when we use uniform distributions instead of exponential distributions?

A Fundamental Result in Queueing Theory

$M/G/1$ queue

- Exponential interarrival times: "M" = memoryless or Markovian
- Service times have arbitrary distribution: "G" = general
- Single server: "1"
- m_s = mean service time
- σ_s^2 = variance of service time
- $\lambda = 1/(\text{mean interrival time})$ = average arrival rate
- $\rho = \lambda m_s$ = traffic intensity
- W = steady-state waiting time

Pollaczek - Khintchine Formula

$$E[W] = \left(\frac{\rho}{1-\rho}\right)\left(\frac{m_s}{2}\right)\left(1+\frac{\sigma_s^2}{m_s^2}\right)$$

Effect of Variability

- Assume interarrival times: Exponential(5.08)

Service-Time Distn	Mean	Variance	$E[W]$
Unif(3.10, 6.46)	4.78	0.94	39.65
Expo(4.78)	4.78	22.85	76.16

- Pollaczek - Khintchine Formula

$$E[W] = \left(\frac{\rho}{1-\rho}\right)\left(\frac{m_s}{2}\right)\left(1+\frac{\sigma_s^2}{m_s^2}\right)$$

Conclusions

What did we learn?

- **Simulation**
  - Using ARENA to simulate queueing systems.
  - A process-oriented approach to simulation modeling.
  - Simulation outputs can be very sensitive to simulation inputs.
- **Queueing Systems**
  - Pollaczek - Khintchine Formula
  - To accurately describe performance of queuing system, we need:
    - ▷ Mean **and SD** of interarrival and service times.
    - ▷ A very important finding.