Math 211 **Final Exam** December 20, 2006

Read each problem carefully. Please show all your work for each problem! Use only those methods discussed thus far in class. Always simplify when possible. No calculators!

1. (10 points) Find a parametric equation of the line of intersection of the two planes:
$$x - 2y + z = 1, \qquad 3x - 2y + z = 5.$$

2. (10 points) A projectile is launched at $t = 0$ seconds with the initial velocity $\mathbf{v}(0) = 100\mathbf{i} + 100\mathbf{j} + 100\mathbf{k}$ meters/second from the point $\mathbf{r}(0) = 500\mathbf{i} - 200\mathbf{j}$ meters. Assuming that the projectile moves under constant acceleration $\mathbf{a}(t) = -10\,\mathbf{k}$ meters/second^2 due to gravity, determine:
   (a) The velocity $\mathbf{v}(t)$ of the projectile as a function of time.
   (b) The position $\mathbf{r}(t)$ of the projectile as a function of time.
   (c) The position and time at which the projectile will hit the ground (i.e. when $z(t) = 0$).

3. (10 points)
   (a) Find an equation of the tangent plane to the surface $xe^{zy} + 2y^2 - z = 0$ at point $(-2, 1, 0)$.
   (b) Find a parametric equation of the normal to this surface at point $(-2, 1, 0)$.

4. (10 points) Let $f(x, y) = x \cos y + y \cos x$.
   (a) Find the directional derivative of f at point $(\pi/6, \pi/3)$ in the direction $\mathbf{v} = \mathbf{i} + \sqrt{3}\,\mathbf{j}$.
   (b) Find the maximum rate of change of f at point $(\pi/6, \pi/3)$.

5. (10 points) Find and classify all the critical points of the function
$$f(x, y) = 2x^2 + 5xy + 4y^2 - 3x + 5y + 10.$$

6. (10 points) Use Lagrange Multipliers to find the minimum value of the function $f(x, y) = 3x - 4y + 5$ subject to the constraint $x^2 + y^2 = 4$.

7. (10 points) Evaluate the double integral
$$\iint_D xe^{2y}\,dA,$$
where D is bounded by $y = 0$, $y = x^2$, $x = 2$.

8. (10 points) Use triple integral to compute the volume of a solid enclosed by the surfaces $x^2 + y^2 = 1$, $z = 0$, $z = 3 - x + y$. (*Hint:* use cylindrical coordinates).

9. (10 points) Use the Fundamental Theorem of line integrals to compute $\int_C \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F} = e^x \cos y\,\mathbf{i} - (e^x \sin y + 3)\,\mathbf{j}$, and C is any path that starts at $(0, \pi)$ and ends at $(\pi, 0)$:
   (a) First determine that the vector field $\mathbf{F}$ is conservative.
   (b) Then find the potential function.
   (c) Use the result of part (b) to compute the integral.

10. (10 points) Use Green's Theorem to evaluate the integral $\oint_C y\,dx - x\,dy$, if C consists of the straight line segment connecting $(0, 0)$ to $(0, 2)$, followed by the segment of the parabola $y = 2 - x^2$, and then the straight line segment from $(1, 1)$ to $(0, 0)$.