Math 331, **Midterm Examination**, Thursday October 12, 2006

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1) Consider the following Initial Boundary-Value Problem for the heat equation with non-homogeneous boundary conditions:

$$\begin{aligned} &u_t = 2u_{xx} \quad ; \quad 0 < x < L,\ t > 0 \\ &u(0,t) = 10 \quad ; \quad t > 0 \\ &u_x(L,t) = 1 \quad ; \quad t > 0 \\ &u(x,0) = 2x + 10 \quad ; \quad 0 < x < L \end{aligned}$$

- Find the equilibrium temperature distribution.
- Use the method of separation of variables to find the solution $u(x,t)$ of the above initial boundary-value problem.

2) Use the method of separation of variables to solve the following Initial Boundary-Value Problem for the heat equation with a heat-energy sink proportional to the temperature:

$$\begin{aligned} &u_t = u_{xx} - u \quad ; \quad 0 < x < L,\ t > 0 \\ &u_x(0,t) = 0 \quad ; \quad t > 0 \\ &u_x(L,t) = 0 \quad ; \quad t > 0 \\ &u(x,0) = f(x) \quad ; \quad 0 < x < L \end{aligned}$$

What is the temperature as $t \to \infty$?

3) Solve Laplace's equation in the rectangle $(x,y) \in [0,L] \times [0,H]$ with the following boundary conditions:

$$\begin{aligned} &u(x,0) = 0 \quad ; \quad 0 < x < L \\ &u(x,H) = g(x) \quad ; \quad 0 < x < L \\ &u(0,y) = 0 \quad ; \quad 0 < y < H \\ &u(L,y) = f(y) \quad ; \quad 0 < y < H \end{aligned}$$

Sketching the domain and the boundary conditions may help you decide how to go about doing this problem.