max possible: 80 ①

Midterm Solutions, MATH 331, f'06

#1] a) Equilibrium solution satisfies

$$\left.\begin{aligned} &u^{eq}_{xx}(x)=0 \quad 0<x<L \\ &u^{eq}(0)=10 \\ &u^{eq\,\prime}(L)=1 \end{aligned}\right\} \begin{aligned} &u^{eq}(x)=c_1+c_2x \\ &\Rightarrow u^{eq}(0)=10 \text{ implies } c_1=10 \\ &u^{eq\,\prime}(L)=1 \text{ implies } c_2=1 \end{aligned}$$

So $u^{eq}(x)=x+10$ (10)

b) To solve the IBVP as given we must employ the $u^{eq}(x)$ to turn the problem into one with homogeneous boundary conditions:

Let $u(x,t)=v(x,t)+u^{eq}(x)$. Then v satisfies

$v_t=2v_{xx}$; $0<x<L$, $t>0$

The boundary conditions on u imply for v:

$u(0,t)=10=v(0,t)+u^{eq}(0)=v(0,t)+10 \Rightarrow v(0,t)=0$

(10) $u_x(L,t)=1=v_x(L,t)+u^{eq\,\prime}(L)=v_x(L,t)+1 \Rightarrow v_x(L,t)=0$

Now the initial condition:

$u(x,0)=2x+10=v(x,0)+x+10 \Rightarrow v(x,0)=x$.

The method of separation of variables can