now be used to find $V(x,t)$ $\Rightarrow V(x,t) = \phi(x) \cdot h(t)$

$$\Rightarrow \frac{h'(t)}{2h(t)} = \frac{\phi''(x)}{\phi(x)} = -\lambda$$

$\phi'' + \lambda\phi = 0 \; ; \; 0 < x < L$

$\phi(0) = 0$

$\phi'(L) = 0$

$h' + 2\lambda h = 0 \; ; \; t > 0$

first we have to determine the ϕ's and the λ's allowed by the boundary conditions (the eigenfunctions and the eigenvalues):

Checking for $\lambda < 0$ and $\lambda = 0$ we find none.

$\lambda > 0 \Rightarrow \phi(x) = C_1 \cos\sqrt{\lambda}x + C_2 \sin\sqrt{\lambda}x$

$\phi(0) = 0 \Rightarrow C_1 = 0$, $\phi(x) = C_2 \sin\sqrt{\lambda}x$

$\phi'(L) = 0 \Rightarrow C_2\sqrt{\lambda}\cos(\sqrt{\lambda}L) = 0 \Rightarrow$

$\cos(\sqrt{\lambda}L) = 0$ is the eigenvalue eqtn.

$\sqrt{\lambda}L \to \sqrt{\lambda_n}L = \left(n + \frac{1}{2}\right)\pi \; ; \; n = 0, 1, 2, \ldots$ or

$= (2n-1)\frac{\pi}{2} \; ; \; n = 1, 2, \ldots$ (same as above)

$$\Rightarrow \lambda_n = \frac{\left(n+\frac{1}{2}\right)^2 \pi^2}{L^2} \quad \text{and} \quad \phi_n(x) = \sin\left[\frac{\left(n+\frac{1}{2}\right)\pi}{L} x\right]$$

$n = 0, 1, 2, \ldots$