\( \lambda_n = \frac{n^2 \pi^2}{L^2} \), \( \phi_n(x) = \cos\left(\frac{n\pi x}{L}\right) \) \( n = 1, 2, \ldots \)

and \( h(t) = h_n(t) = e^{-(1 + \frac{n^2 \pi^2}{L^2}) t} \) \( \forall n = 0, 1, 2, \ldots \) so

The general solution is

\[
U(x,t) = e^{\Delta_0} + \sum_{n=1}^{\infty} A_n \cos\left(\frac{n\pi x}{L}\right) e^{-\left(1 + \frac{n^2 \pi^2}{L^2}\right)t}
\]

where

\( A_0 = \frac{1}{L} \int_0^L f(x) dx \) and \( A_n = \frac{2}{L} \int_0^L f(x) \cos\left(\frac{n\pi x}{L}\right) dx \)

to satisfy the initial condition.

Also, \( U(x,t) \to 0 \) although we have two no-flux boundary conditions.

That's ok since although heat energy can't escape out the ends at \( x=0, L \) we have a sink of heat energy \(-U\) in the problem.