Math 331 • Midterm Examination • Victor Matveev • Spring 2018

Please read the assignment carefully, and show all work. No notes or electronic devices allowed.

1. **(20pts, 14min)** Consider the function $f(x)=\begin{cases} x, & 0<x<\frac{1}{2} \\ 0, & \frac{1}{2}<x<1 \end{cases}$ defined on the interval $0<x<1$

   a) Write down the partial sum of first 4 non-zero terms in the **sine** series of $f(x)$ (given ***L*=1**). To avoid mistakes, calculate even and odd terms *separately*.

   b) Graph the odd periodic extension of $f(x)$ and the first non-zero term in the sine series for $-3<x<3$

2. **(2×30pts, 2×25min)** Solve the following two partial differential equations, *explaining all steps in the solution.* When solving the homogeneous boundary value problem, make sure to consider all signs of λ, and sketch any two solutions (e.g. ϕ_1 and ϕ_2). **Check** your solution if in doubt.

(A) $$\begin{cases} \dfrac{\partial^2 u}{\partial t^2}=c^2\dfrac{\partial^2 u}{\partial x^2} & (0<x<1;\ t>0) \\ u(0,t)=0;\ \dfrac{\partial u}{\partial x}(1,t)=0 \\ u(x,0)=1;\ \dfrac{\partial u}{\partial t}(x,0)=0 \end{cases}$$

(B) $$\begin{cases} \dfrac{\partial^2 u}{\partial x^2}+\dfrac{\partial^2 u}{\partial y^2}=0 & (0<x<L,\ \ 0<y<H) \\ \dfrac{\partial u}{\partial x}(0,y)=0;\ \dfrac{\partial u}{\partial x}(L,y)=g(y) \\ \dfrac{\partial u}{\partial y}(x,0)=0;\ u(x,H)=0 \end{cases}$$

3. **(10pts, 8min)** Find the ***equilibrium*** solution of the following heat equation in a 4-dimensional symmetric region enclosed between concentric hyperspheres with inner radius of 1 and outer radius of *R*>1. Where does the energy enter, and where does it leave this region? Assume $T_o>0$.

$$\begin{cases} \dfrac{\partial u}{\partial t}(r,t)=\dfrac{1}{r^3}\dfrac{\partial}{\partial r}\left[r^3\dfrac{\partial u}{\partial r}\right] & (1<r<R,\ \ t>0) \\ u(1,t)=0; \quad u(R,t)=T_o \\ u(r,0)=T_o\ln(r/R) \end{cases}$$

4. **(10pts, 8min)** Solve these ordinary differential equations (note: boundary conditions are *not* homogeneous):

$$(a)\ \begin{cases} \dfrac{d^2g}{dx^2}=9g(x) \ \ (0<x<1) \\ g(0)=1 \\ g(1)=0 \end{cases} \qquad (b)\ \begin{cases} x^2\dfrac{d^2g}{dx^2}=g(x) \ \ (x>0) \\ g(0)=0 \\ \dfrac{dg}{dx}(0)=1 \end{cases}$$