Math 331 • Midterm Examination • Victor Matveev • Spring 2019

Please read the assignment carefully, and show all work. Please check your answers!

1. **(20pts, 15min)** Consider the function $f(x) = \cos(x/2)$ defined within $0 < x < \pi$ (note: $\boldsymbol{L = \pi}$)

   a) Write down the **cosine** series for $f(x)$, as well as the partial sum of first 3 non-zero terms (counting A_0 as the first term). Hint: use the identity $2\cos\alpha\cos\beta = \cos(\alpha+\beta) + \cos(\alpha-\beta)$

   b) On the interval [–3π, 3π], plot the **even** periodic extension of *f*(*x*), and the sum of first two non-zero term in the series (i.e. sum of terms corresponding to coefficients A_0 and A_1)

2. **(35pts, 25min)** Solve this vibrating string equation with a restoring force, *explaining all steps.* When separating variables, move the constant γ to the time-dependent part so that the BVP is in the standard form φ''= –λ φ(*x*). **If you solve this problem with γ = 0, you will get 25 points instead of 35 points.**

$$\begin{cases} \dfrac{\partial^2 u}{\partial t^2} = c^2 \dfrac{\partial^2 u}{\partial x^2} - \gamma u \quad (0 < x < L, \ \ t > 0; \ \ \gamma = \mathbf{const}) \\ \dfrac{\partial u}{\partial x}(0,t) = 0; \ \ u(L,t) = 0 \\ u(x,0) = f(x); \quad \dfrac{\partial u}{\partial t}(x,0) = 0 \end{cases}$$

3. **(15pts, 10min)** Find the ***equilibrium*** solution of the following heat equation in a volume enclosed between two long co-axial cylinders with fixed temperatures on the inner and outer surfaces. Where does the heat enter this volume, and where does it leave? (Hint: ln(2) ≈ 0.693, although that's not important)

$$\begin{cases} \dfrac{\partial u}{\partial t}(r,t) = \dfrac{1}{r}\dfrac{\partial}{\partial r}\left(r\dfrac{\partial u}{\partial r}\right) - \dfrac{1}{r} \quad (1 < r < 2, \ \ t > 0) \\ u(1,t) = 3; \quad u(2,t) = 0 \end{cases}$$

4. **(10pts, 10min)** Separate variables in the following differential equation by plugging in $u(x,y) = g(x)h(y)$, and write down (but do *not* solve) the two equations for g(*x*) and *h*(*y*) that you obtain:

$$\frac{x}{y}\frac{\partial}{\partial y}\left(y^2\frac{\partial u}{\partial y}\right) = y\frac{\partial^2 u}{\partial x^2}$$

5. **(10pts, 10min)** Does the function *u*(*x*,*y*) below satisfy this Laplace's equation? Explain your answer.

$$\begin{cases} u_{xx} + u_{yy} = 0 \quad (0 < x < L, \, 0 < y < H) \\ u(0,y) = u_x(L,y) = 0 \\ u(x,0) = f(x); \quad u_y(x,H) = 0 \end{cases} \qquad u(x,y) = \sum_{\substack{m=1,3,5.. \\ \textbf{Odd m}}}^{\infty} C_m \sin\frac{m\pi x}{2L}\cosh\frac{m\pi y}{2L}$$

6. **(10pts, 10min)** Solve the following initial value problem to find $f(x)$. Hint: we solved this type of equation quite recently. Make sure to check your solution!

$$\begin{cases} 2x\,f'(x) - 3f(x) = 0 \quad (x > 1) \\ f(1) = 1 \end{cases}$$