1) (24pts) For each, find all distinct values of \( z \) in Cartesian form, and show roughly their locations as points in the complex plane:

(a) \( z = (8 - 8i)^{2/3} \)  
(b) \( z = 2^i \)  
(c) \( \sin z = i \)  

2) (12pts) Sketch the region \( 1 \leq |z| \leq e, \ \text{Re} \ z > 0 \), and sketch its image under the transformation \( w = \log(z) \).  

3) (14pts) Is function \( f(z) = \text{Im} \ z \) continuous at \( z = 0 \)? Is it differentiable at \( z = 0 \)? Explain your answers; use two methods to analyze differentiability: (1) the limit definition of derivative, (2) the Cauchy-Riemann equations.  

4) (15pts) Show that the function \( u(x, y) = e^{-2y} \cos(2x) \) is harmonic; find its harmonic conjugate, \( v(x, y) \), and express the function \( f = u(x, y) + iv(x, y) \) in terms of \( z \).  

5) (14pts) Use curve parametrization (instead of an anti-derivative or Cauchy theorem) to integrate \( \oint \frac{dz}{z^2} \) around the closed contour \( \Gamma \) shown in the figure below. Check your answer by comparing with the exact value of this integral.  

6) (21pts) Which of these integrals equal zero for any loop \( \Gamma \) contained within the domain \( |z| < 1 \)? Explain your answers  

\[
\begin{align*}
\text{(a)} \quad & \oint_{\Gamma} \frac{dz}{(2z+i)^{3/2}} \\
\text{(b)} \quad & \oint_{\Gamma} \frac{dz}{\cos z} \\
\text{(c)} \quad & \oint_{\Gamma} \frac{1}{\log \left( \frac{z+1}{2} \right)} \, dz
\end{align*}
\]

Alternative to problem 2  
(note the smaller number of points):  

2') (10pts) Sketch the region \( 1 \leq |z| \leq 2, \ \text{Re} \ z < 0, \ \text{Im} \ z > 0 \), and sketch its image under the transformation \( w = \frac{i}{z^2} \).