

Math 332-001 First Midterm

February 14, 2009

1. For the statements below, provide a proof if true or a counterexample if false:

$$(a) \quad \overline{\left(\frac{z_1}{z_2 z_3}\right)} = \overline{z_1} \left(\frac{1}{\overline{z_2 z_3}}\right) \quad (b) \quad |z_1 + z_2|^2 + |z_1 - z_2|^2 = 2(|z_1|^2 + |z_2|^2)$$

2. Find all values of $(1 - i)^{1/3}$. Give your answers in *cartesian* form.
3. Identify the image of the region defined by $z = x + iy$ with

$$x^2 + y^2 \leq 1 \quad \text{and} \quad y > 0$$

under the transformation $f(z) = 1/z$. Sketch both regions (i.e., the original region and its image).

4. Prove, using the δ - ϵ definition of the limit, that the function $f(z) = 2\bar{z}$ is continuous at all points $z_0 \in \mathbb{C}$. Prove, using the method of your choice, that $f(z)$ is nowhere differentiable.
5. Show that $u(x, y) = \sin x \sinh y$ is harmonic and find its harmonic conjugate $v(x, y)$. Find $f(z) = u + iv$ explicitly as a function of z . Is it true that $f(z) = f(\bar{z})$? What is the largest open, connected set on which $1/f(z)$ is analytic?
6. Find *all* values for the following, then identify their principal values:

$$(a) \quad \log(2i) \quad (b) \quad \arccos(2 + 3i)$$