Math 332-001 Second Midterm

February 14, 2009

Answer all questions in the booklet provided. Each carries equal weight.

1. Let C_1 be the portion of the unit circle running in a counter-clockwise direction from $(1,0)$ to $(1/\sqrt{2}, 1/\sqrt{2})$, let C_2 be the straight line running from $(1/\sqrt{2}, 1/\sqrt{2})$ to $(0,0)$, and let C_3 be the straight line running from $(0,0)$ to $(1,0)$. Let $C = C_1 + C_2 + C_3$. If $f(z) = \overline{z}$, calculate

$$(a) \int_{C_1} f(z)\, dz, \quad (b) \int_{C_2} f(z)\, dz, \quad (c) \int_{C_3} f(z)\, dz, \quad (d) \int_{C} f(z)\, dz.$$

   Without doing any calculations, provide a different function $f(z)$ that would give the same answer you calculated in part (c) and an answer of zero in part (d). Explain your answer.

2. Let C be the contour running from $(0,-1)$ to $(0,1)$ on the unit semicircle contained in the left half-plane. Use antiderivatives to find $\int_C f(z)\, dz$ where

$$(a) \quad f(z) = ze^z, \qquad (b) \quad f(z) = \frac{1}{z}.$$

   For which of the above integrals would the answer change if the contour were connecting the same two points but were lying on the unit semicircle in the *right* half-plane? Explain.

3. Use either the Cauchy-Goursat Theorem or the Cauchy Integral Formula to evaluate the following integrals, where C is the positively oriented closed contour lying on the unit circle.

$$(a) \quad \int_C \frac{\cosh z}{(z-3)(2z-i)}\, dz \qquad (b) \quad \int_C \frac{e^z}{z^3}\, dz \qquad (c) \quad \int_C \frac{\sinh z}{z}\, dz$$

4. Find a Laurent series that converges to

$$f(z) = \frac{1}{z^2 - 4z}$$

   in an annular domain centered at $z = 1$ and containing the point $z = 2+2i$. State where the Laurent series converges.