1. Consider a sphere of radius $a$ with charge density increasing with distance from the center as $\rho(r) = \alpha r$, where $\alpha$ is some constant. Find the electric potential $\Phi$ both inside and outside of the sphere, by integrating the Poisson’s equation in spherical coordinates ($\Delta \Phi = -\rho/\varepsilon_0$ inside the sphere, and $\Delta \Phi = 0$ outside of the sphere), as we did in class. Assume that the solution depends on $r$ only: $\Phi = \Phi(r)$. Note that the electric field should be continuous across the surface of the sphere; this condition will fix the integration constants.

2. Check by differentiation that the potential inside and outside the sphere that you found in problem 1 satisfies the Poisson’s equation re-written in Cartesian coordinates:

$$\Delta \Phi^\text{in} = \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) \Phi^\text{in} = -\frac{\alpha r}{\varepsilon_0} = -\frac{\alpha \sqrt{x^2 + y^2 + z^2}}{\varepsilon_0} \quad \text{- Inside the sphere}$$

$$\Delta \Phi^\text{out} = \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) \Phi^\text{out} = 0 \quad \text{- Outside the sphere}$$

3. Consider a very long cable (cylinder) of radius $a$ with a constant charge density inside the cable, $\rho$, and zero charge density outside the cable. Use cylindrical coordinates to find the electric potential $\Phi$ and the radial component of electric field $E_R$ both inside and outside of the cable, by solving (integrating) $\nabla^2 \Phi = -\rho/\varepsilon_0$ inside the cable, and $\nabla^2 \Phi = 0$ outside of the cable, as we did in class for a sphere in spherical coordinates. Assume that $\Phi$ depends only on $R$, the distance from the $z$-axis, which is the axis of the cable: $\Phi = \Phi(R)$. Use Eq. 6.16 for the Laplacian. Assume that $E_R$ is continuous across the surface of the cable; this condition will fix one of the integration constants.

4. Use the divergence theorem instead of the Poisson’s equation to find the electric field inside a uniformly charged sphere in example 8.3 on p. 136: $E_r(r) = \rho r / 3 \varepsilon_0$. Hint: integrate both sides of equation $\nabla \cdot E = \rho / \varepsilon_0$ over the volume of a sphere of radius $b < a$ to obtain $E_r(b) = \rho b / 3 \varepsilon_0$. 