Equivalent circuit of a “passive” cell

Let’s assume only K⁺ channels are present:

\[
\text{Kirchhoff law:} \quad I_C + I_K = I \\
\Rightarrow I = C \frac{dV}{dt} + G_K (V - V_K) \Rightarrow C \frac{dV}{dt} = I - G_K (V - V_K)
\]

Here \( V_K = \frac{RT}{zF} \ln \left( \frac{[K^+]_{\text{out}}}{[K^+]_{\text{in}}} \right) = \frac{k_B T}{q} \ln \left( \frac{[K^+]_{\text{out}}}{[K^+]_{\text{in}}} \right) \) is the potassium reversal potential (Nernst potential).

• NOTE: \( V_K < 0 \), since a very negative potential is required to force K⁺ ions to move into the cell against their concentration gradient. See proof of this expression in another file…

We can easily generalize to two different channel types are present, K⁺ and Cl⁻:

\[
I_C + I_K + I_{Cl} = I \\
\Rightarrow I = C \frac{dV}{dt} + G_K (V - V_K) + G_{Cl} (V - V_{Cl}) \Rightarrow C \frac{dV}{dt} = I - G_K (V - V_K) - G_{Cl} (V - V_{Cl})
\]

Let’s who that the two (or any number) of ionic currents can be combined into a single term:
Total "leak" / "input" conductance:

\[
G_L = G_K + G_{cl}
\]

Where we introduce two quantities:

- "Resting" / "equilibrium" potential:
  \[
  V_r = \frac{G_K V_K + G_{cl} V_{cl}}{G_K + G_{cl}}
  \]
  = average of reversal potentials weighted by their conductances

We may perform another simplification:

\[
C \frac{dV}{dt} = I - G_L (V - V_R)
\]

\[
= -G_L \left( V - V_R - \frac{I}{G_L} \right)
\]

Now let's denote \( \frac{1}{G_L} = R_L \) ("Input resistance")

\[
= -G_L \left( V - (V_R + R_L I) \right)
\]

\[
= -G_L (V - V_{RI}) \quad \text{where equilibrium potential in non-zero current is} \quad V_{RI} = V_R + R_L I
\]

Finally, we can divide both sides of the equation by C, and note that \( G_L / C = 1 / (R_L C) = 1 / \tau_m \)

\[
\frac{dV}{dt} = -\frac{V - V_{RI}}{\tau_m}
\]

Denoting the deviation from equilibrium as \( v = V - V_{RI} \), and noting that \( dv = dV \), we obtain:

\[
\frac{dv}{dt} = -\frac{v}{\tau_m} \quad \Rightarrow \quad v(t) = v_o e^{-t/\tau_m} \quad \Rightarrow \quad V(t) - V_{RI} = (V_o - V_{RI}) e^{-t/\tau_m}
\]

\[
\Rightarrow \quad V(t) = V_{RI} + (V_o - V_{RI}) e^{-t/\tau_m} \quad \text{potential decays exponentially to} \ V_{RI} \ \text{with characteristic time} \ \tau_m