Math 613 * Fall 2019 * Victor Matveev * Homework 5

1. Consider the function $f(x) = \int_0^\infty e^{-t} \ln(1 + x t)\, dt$

a) Obtain the asymptotic expansion of this function for x → 0 using the familiar Taylor series expansion of the logarithmic function, and then taking the integral term-by-term. Use the gamma function identity $\int_0^\infty t^k e^{-t} dt = \Gamma(k+1) = k!$

b) Did you obtain a Taylor series or an asymptotic series for $f(x)$?

c) Take a value of x=**0.1**, and plot the dependence of the partial sum of the asymptotic series, $S_N(x)$, on N, for N from **1 to 20**. What do you think would be the best estimate for $f(\mathbf{0.1})$?

2. Consider the following ODE where ε is a small positive constant parameter:

$$\begin{cases} \frac{dy}{dt} = \cos\sqrt{\varepsilon y} \\ y(0) = 1 \end{cases}$$

a) Make a qualitative sketch of the solution ($y(t)$ vs t) using methods learned in the first week of class.

b) Expand the right-hand side in the Taylor series up to 3rd order in ε, and consider solutions in the form of perturbative series $y(t) = y_o(t) + \varepsilon y_1(t) + \varepsilon^2 y_2(t) + \varepsilon^3 y_3(t) + O(\varepsilon^4)$, with the following initial conditions:

$$y_0(0) = 1, \quad y_{k>0}(0) = 0.$$

c) Plot the partial sums for $y(t)$ from 2 up to 4 terms as a function of t, for ε=0.1, and compare with your qualitative plot in part "a".