

**Math 630-102**  
**Homework 2**

**Section 1.4**

**Problem 13 (a, b, d).** By trial and error find examples...

Note: this means that you are asked to guess the matrix elements, although you may easily find a more systematic way to solve this problem. You don't have to do part *c* since it's somewhat tricky.

**Problem 21.** Find the powers  $A^2$ ,  $A^3$  ( $A^2$  times  $A$ ), and  $B^2$ ,  $B^3$ ,  $C^2$  and  $C^3$ ...

Note: the last question asks you to obtain expressions for  $A^k$ ,  $B^k$  and  $C^k$ , for any  $k$ . This should be obvious once you solve the first part.

**Problem 24.** Which three matrices  $E_{21}$ ,  $E_{31}$ ,  $E_{32}$  put "A" into a triangular form "U"?...

Recall that  $M$  is the inverse of matrix  $L$  in the  $LU$  factorization. Note that subscripts of matrices  $E$  here and elsewhere in the book indicate rows that are operated on ( $E_{21}$  indicates that this matrix adds or subtracts a multiple of row 1 from row 2). However, in class we used subscripts just to indicate the order of operations ( $E_1$  we called our first row operation,  $E_2$  our second operation, etc.)

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**Section 1.5**

**Problem 3.** Apply elimination to produce the factors  $L$  and  $U$  for...

Note: recall that elements of  $L$  contain the inverse coefficients of your row operations. Note also that you can always check your answer by multiplying  $L$  and  $U$  to obtain  $A$ .

**Problem 5.** Factor  $A$  into  $LU$ , and write down the upper triangular system  $Ux = c$  which appears after elimination, for...

**Problem 9, part (b) only:** Solve the system  $Ax = b$  starting with  $Lc = b$

Note: this is where you do the step 2 of the method described in class. In part (a) the matrix  $A$  is already decomposed into  $LDU$ . Once you obtain  $c$ , solve  $DUx = c$ .

**Problem 11.** Solve as two triangular systems, without multiplying  $LU$  to find  $A$ ....

Note: again, do the step 2 of the method described in class. Recall that first you solve  $Lc=b$  (where  $b$  is the right-hand side column vector), and once you know  $c$ , you can then solve  $Ux=c$

**Problem 15.** Find the PA = LDA factorizations (and check them) for...

Note: you have to do any necessary row exchanges first (P), even though it requires you to start the factorization to see if a row exchange is indeed needed. In practice of course computer algorithms will avoid repeating the same steps twice.

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### Section 1.6

**Problem 1.** Find the inverses (no special system is required) of...

Note: “no special system is required” means that you can guess the elements by trial and error. Hint for the last matrix:  $\sin^2\theta + \cos^2\theta = 1$

**Problem 3.** From  $AB=C$  find the formula for  $A^{-1}$ . Also find  $A^{-1}$  from  $PA = LU$ .

Hint: use the rule for the inverse of a product, and do a simple linear algebra step.

**Problem 13.** If  $A = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$  and  $B = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$ , compute  $A^T B$ ,  $B^T A$ , and  $BA^T$