Refrigeration & Heating
(vapor-compression cycle)

1. Basic Thermodynamics

- Phase diagram of refrigerants.

**FIGURE 7.1**
P-v-T surface of a substance that contracts on freezing.

**FIGURE 7.2**
P-v-T surface of a substance that expands on freezing (like water).

- Five phases:
gas, liquid, solid, supercritical, plasma

- Ideal gas approximation for "vapors".
• Cycles

(1) Engine cycle: power generation!
(at the expense of heat transfer from $T_H$ to $T_L$.)

- Power generated:

$$W_{net} = \int p\,dv = \int_{v_i}^{v_f} p\,dv - \int_{v_f}^{v_i} p\,dv$$

power generated in $\text{A}$

power needed in $\text{B}$ to make a cycle!

- Energy consumption (from $T_H$)

$$\dot{Q}_H = \int_{A_s_1}^{s_2} T\,ds$$

$\{\text{fuel combustion, nuclear reaction, other } T_H \text{ sources}\}$

- Waste heat dumped (to $T_L$)

$$\dot{Q}_L = \int_{s_1}^{s_2} T\,ds$$
- Energy utilization
  * Efficiency \( \eta = \frac{W}{Q_H} \):
    (energy conversion)
  * 2nd law efficiency \( \eta_{II} = \frac{\eta}{\eta_{cannot}} \)
    \[
    \eta_{II} = \frac{W}{Q_H \left(1 - \frac{T_{min}}{T_{max}}\right)}
    \]
    (effectiveness of maximum possible energy conversion)
(2) Reversed cycle:
  heat generation from \( T_1 \) to \( T_H \)!
  (at the expense of power consumption)

- Refrigeration (\( Q_L \) from \( T_L \))
  \[
  \dot{Q}_L = \int_{S_i}^{S_f} T \, ds
  \]
- Heating: heat pump ($Q_H$ to $T_H$)

$$Q_H = \int_{S_i}^{S_f} T \, ds$$

- Power consumption:

$$W = \int p \, dv = \int_{A}^{B} p \, dv$$

- COP of Refrigeration

Coefficient of performance

$$\text{COP}_R \equiv \frac{\dot{Q}_L}{W}$$

$$\text{COP}_R \equiv \frac{\text{COP}_R}{\text{COP}_{	ext{carnot}, R}} = \frac{\text{COP}_R (T_{\text{max}} - T_{\text{min}})}{T_{\text{min}}}$$

- COP of Heat pump

$$\text{COP}_{HP} \equiv \frac{\dot{Q}_H}{W}$$

$$\text{COP}_{II, HP} \equiv \frac{\text{COP}_{HP}}{\text{COP}_{	ext{carnot}, HP}} = \frac{\text{COP}_{HP} (T_{\text{max}} - T_{\text{min}})}{T_{\text{max}}}$$
2. Vapor-compression Cycle

![Diagram of a vapor-compression cycle]

Figure 7.3 Schematic view of room air conditioner.

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**Idealized Cycle**

\[
\dot{Q}_L = m_R \cdot \Delta h_A \\
\dot{Q}_H = m_R \cdot \Delta h_C \\
\dot{W} = m_R \cdot \Delta h_B
\]
- realistic cycle

1. heat leak (throttling)
2. pressure drop (evaporator)
3. superheating (compressor inlet)
4. \( \eta \neq 1 \) (compressor)
5. pressure drop (cooling coil & condenser)
6. subcooling

* — more analysis in the following:

— flow and heat transfer considerations

\[ \dot{m}_L = \frac{P_L}{h_L} \left( T_{RA} - T_L \right) = m_R \Delta h_L \]
\[ \Delta P_L = f(m_R, A_L) \]
2. \( \dot{Q}_H = \dot{m}_H A_H (\bar{T}_H - T_\infty) = \dot{m}_R \Delta h_H \) \\
\( \Delta P_H = f(\dot{m}_R, A_H) \) \\
material limit

3. \( \eta_{\text{compressor}} = \frac{\dot{W}}{\dot{W}_s} \) \( \Rightarrow (P_{HH}, T_{max}) \)

4. \( \dot{h}_L, \dot{h}_H, A_L, A_H \) \( \Rightarrow \) \\
\( \{ \text{Fin design} \} \) \\
\( \{ \text{Fan/pump} \} \)

3. Application (Psychrometric cycles)
   - Cooling (air conditioning)
     - Ideal cycle

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\[ \dot{Q}_H = \dot{m}_H A_H (\bar{T}_H - T_\infty) = \dot{m}_R \Delta h_H \]
\( \Delta P_H = f(\dot{m}_R, A_H) \)
material limit

\( \eta_{\text{compressor}} = \frac{\dot{W}}{\dot{W}_s} \) \( \Rightarrow (P_{HH}, T_{max}) \)

\( \dot{h}_L, \dot{h}_H, A_L, A_H \) \( \Rightarrow \) \\
\( \{ \text{Fin design} \} \) \\
\( \{ \text{Fan/pump} \} \)
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\[ \dot{Q}_c = \dot{Q}_L = m_a \cdot \Delta h_{PA} \]

\[ \dot{m}_w = m_a \cdot \Delta h_{PA} \]

\[ m_a = \frac{\dot{q}}{\Delta h_{RA}} \]

--- realistic cycle ---

--- Diagram ---

EA --\( \dot{q}_{\text{leakage}} \)-- RA

RA' -- SA' -- Fan Filter

SA -- another room

OA -- A/C -- Room

A by-pass

m_w
realistic factors:

1. Fan \( \Rightarrow (\dot{Q}, \dot{m}_a, \dot{m}_w) \)
2. Filter
3. By-pass \( \Rightarrow R < 1 \) (flow mixing)
4. \( \dot{Q}_{leakage} \) (pipeline insulation)
5. Supply to multiple rooms

\( \Rightarrow \) duct-pipeline system design

* Homework:

How above factors affect \( (\dot{Q}_c, \dot{m}_w) \)? Explain using "realistic" psychrometric cycle.

- Heating (heat-pump)
  - ideal cycle

\[ \text{Room} \rightarrow m_w \]
\[ \text{H.P.} \rightarrow \dot{Q}_H \]
\[ \text{Humidifier} \]
\[ \text{OA} \rightarrow \text{RA} \rightarrow \text{EA} \]
\[ \text{OA} \rightarrow \text{MA} \rightarrow \text{SA} \]
\[ \dot{Q}_{HP} = \dot{Q}_H = \dot{m}_a \Delta h_{HP} \]

\[ \dot{m}_a = \frac{\dot{Q}}{\Delta h_{RA}} \]

- realistic cycle

Homework:

How these factors affect requirement of H.P.?

Explain using "realistic" psychrometric cycle.

Assuming the same humidification process.